

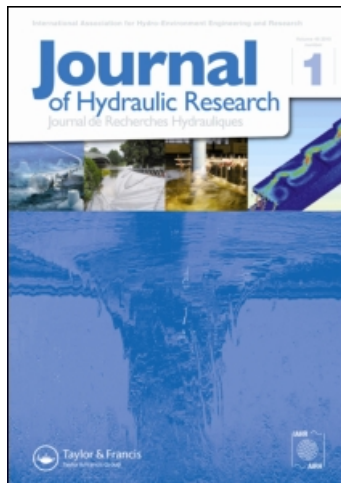
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A depth-integrated model for suspended sediment transport

Un modèle de transport solide en suspension intégré sur la verticale



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SUMMARY

A model for suspended sediment transport in unsteady and non-uniform flow is derived, in which the vertical dimension is eliminated by means of an asymptotic solution. The resulting depth-integrated model is tested for unidirectional flow cases. Also, an application is given to the siltation of a dredged channel.

RÉSUMÉ

Un modèle mathématique de transport solide en suspension en régime non permanent et non uniforme a été élaboré, la dimension verticale ayant été éliminée grâce au recours à une solution asymptotique. Le code de calcul a été essayé pour des cas d'écoulement filaire. De plus, un exemple est fourni pour l'envasement d'un chenal dragué.

1 Introduction

The process of suspended sediment transport in rivers and estuaries is a three-dimensional one. Some models have already been derived which take the vertical direction explicitly into account (Delft Hydraulics Laboratory, 1980, Smith and O'Connor, 1977) by considering a 2-d vertical plane. Extension of these methods to 3-d situations, however, will be very costly, particularly if it is realized that long periods of time may have to be covered. Similar problems for dissolved substances have been solved by introducing the concept of dispersion through depth integration (e.g. Daubert, 1974). This paper analyses the possibility of using such concepts for suspended sediment.

The mass-balance for suspended sediment in a flowing stream can be expressed in the form of a partial differential equation describing the processes of convection, turbulent diffusion and precipitation in terms of the local sediment concentration. If the mass-balance equation is depth averaged, the process of vertical readjustment of concentration profiles, arguably the most important mechanism involved, is obscured and replaced by an empirical or semi-empirical entrainment function. The necessity to calibrate this very important effect will restrict the predictive power of such depth-averaged models. The verification of an entrainment function usually has to be done indirectly. Thus there is a need to develop other approximate solutions for the mass-balance equation based on more explicit, easily verifiable assumptions.

In this paper the adjustment of the concentration distribution is formulated in terms of similarity

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profiles, including the deviation from local equilibrium. The coefficients of the similarity profiles are shown to depend on the depth averaged concentration \bar{c} and its horizontal derivatives. Thus, for a vertically 2-d situation, the equations to be solved are reduced to 1-d, and for 3-d situations the equations have to be solved in a (horizontally) 2-d region. Only the former case is discussed in this paper. The resulting model can be used together with the depth-averaged hydrodynamic equations in large regions. As usual, the similarity solution cannot be applied for small-scale (near-field) phenomena, but only at sufficiently large scales. Some considerations on these scales are also given in the paper.

2 Scales and magnitudes

The basis for this study is the commonly accepted mass-balance equation for a uniform suspended sediment of fall velocity w_s in a two-dimensional flow field as shown in Fig. 1

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + w \frac{\partial c}{\partial z} = w_s \frac{\partial c}{\partial z} + \frac{\partial}{\partial z} \left(\varepsilon \frac{\partial c}{\partial z} \right) \quad (1)$$

where

- c = the sediment concentration
- u and w = the velocity components in the horizontal and vertical directions
- x and z = the horizontal and vertical coordinates
- t = time
- ε = the turbulent diffusion coefficient for sediment transfer in the vertical direction

Horizontal turbulent diffusion has been assumed to be negligible, which is acceptable at sufficiently large scales.

At the free surface ($z = z_b + a$ where z_b is the bottom level and a the water depth) the vertical sediment flux should be zero:

$$w_s c + \varepsilon \frac{\partial c}{\partial z} = 0 \quad (2)$$

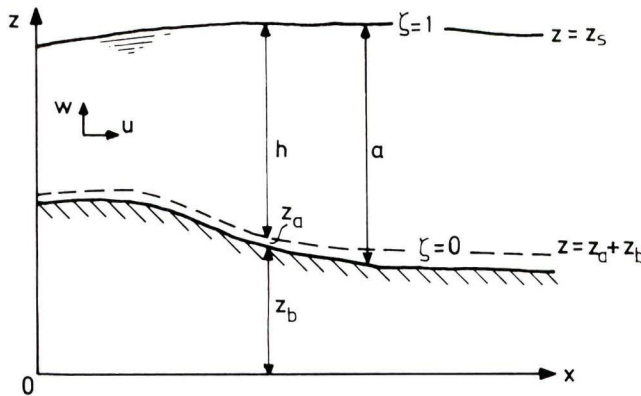


Fig. 1. Definition sketch of two-dimensional field.
Schéma de définition dans le plan vertical.

The lower boundary condition is commonly applied at a height z_a above the bed. Suspended sediment transport will be defined as the transport of sand above the level $z = z_a + z_b$. If there is any transport below this level, this will be defined to be bed load, which is not described by the present theory and has to be added as a separate process. Thus the suspended sediment will be transported over a depth h given by

$$h = a - z_a$$

where a is the flow depth.

If the near-bed concentration (at $z = z_a + z_b$) can be specified in terms of the local flow and sediment parameters, the bed boundary condition that has to be applied is

$$c = c_a \quad \text{at} \quad z = z_a + z_b \quad (3)$$

The specification of the near-bed concentration c_a is a standing problem which is not addressed in this paper. It should be realized that this boundary condition is a crucial factor in any model for suspended sediment transport, including the present one. Various possibilities for the bed boundary condition can be used, e.g. an empirical formulation in terms of the local bed shear stress, or the assumption that c_a corresponds to the equilibrium concentration distribution that would exist under the local flow conditions (i.e. very near the bed, the concentration adjusts immediately to local equilibrium whereas higher in the vertical a slower adjustment occurs). Different expressions have been suggested for the profile of ε in open channel flow (e.g. Rouse, 1937, DHL, 1980). These expressions could in general be reduced to the form (see Galappatti, 1983)

$$\frac{\varepsilon}{w_s h} = \varepsilon' = \varepsilon' \left(\zeta, \frac{w_s}{u_*}, \frac{z_a}{a} \right) \quad (4)$$

where the dimensionless vertical coordinate ζ is defined in equation (14).

Let the flow under consideration be characterized by the horizontal length and velocity scales L and U and vertical length and velocity scales H and UH/L respectively. Let E represent a scale for the turbulent diffusion coefficient ε . Equation (1) could now be written as

$$\frac{H}{w_s T} \frac{\partial c}{\partial t'} + \frac{HU}{Lw_s} \left(u' \frac{\partial c}{\partial x'} + w' \frac{\partial c}{\partial z'} \right) = \frac{\partial c}{\partial z'} + \frac{E}{w_s H} \frac{\partial}{\partial z'} \left(E' \frac{\partial c}{\partial z'} \right) \quad (5)$$

where all quantities marked with (') have been made dimensionless using the corresponding scale ($u = Uu'$, $z = Hz'$, $\varepsilon = EE'$ etc.).

The order of magnitude of E is $\frac{1}{4}\kappa u_* H$ where κ is Von Karman's constant and u_* the shear velocity. Thus,

$$\frac{E}{w_s H} \sim \frac{1}{4} \frac{\kappa u_* H}{w_s H} \sim 0.005 \frac{U}{w_s}$$

which can very well be of the order 1.

Therefore both terms on the right hand side of (5) are of $O(1)$ and are responsible for the vertical readjustment of concentration profiles. The magnitudes of the terms on the left hand side depend on the values of $H/w_s T$ and UH/Lw_s . If these parameters are small, then it is possible to construct an asymptotic solution to (5).

Consider two possibilities.

$$\begin{aligned}\text{Case (A)} \quad UH/Lw_s &= \delta \ll 1 \\ H/w_s T &= \delta \ll 1\end{aligned}$$

$$\begin{aligned}\text{Case (B)} \quad UH/Lw_s &= \delta \ll 1 \\ H/w_s T &= \delta^2\end{aligned}$$

Case (B) implies that

$$T \sim \frac{w_s}{H} \frac{L^2}{U^2} \sim \frac{EL^2}{H^2 U^2}$$

which corresponds to assumptions made by Daubert (1974). As it is possible to show that case (A) includes case (B) and that the only change due to case (B) is that time-derivative terms occur only in the 2nd and higher order solutions, this analysis will be restricted to case (A) only. More details of case (B) are given by Galappatti (1983).

3 Asymptotic solution

Using the definition of δ , it is possible to write (5) as

$$\delta \left(\frac{\partial c}{\partial t'} + u' \frac{\partial c}{\partial x'} + w' \frac{\partial c}{\partial z'} \right) = \frac{\partial c}{\partial z'} + \frac{E}{w_s H} \frac{\partial}{\partial z'} \left(E' \frac{\partial c}{\partial z'} \right) \quad (6)$$

which admits a solution of the type

$$c = \sum_{i=0}^n \delta^i \psi_i + 0(\delta^{n+1}) \quad (7)$$

where

$$\frac{\partial \psi_0}{\partial z'} + \frac{E}{w_s H} \frac{\partial}{\partial z'} \left(E' \frac{\partial \psi_0}{\partial z'} \right) = 0 \quad (8)$$

and

$$\frac{\partial \psi_i}{\partial z'} + \frac{E}{w_s H} \frac{\partial}{\partial z'} \left(E' \frac{\partial \psi_i}{\partial z'} \right) = \left(\frac{\partial}{\partial t'} + u' \frac{\partial}{\partial x'} + w' \frac{\partial}{\partial z'} \right) \psi_{i-1} \quad \text{for } i \geq 1 \quad (9)$$

It is possible to absorb δ and revert to original coordinates and transform (8) and (9) to

$$w_s \frac{\partial c_0}{\partial z} + \frac{\partial}{\partial z} \left(\varepsilon \frac{\partial c_0}{\partial z} \right) = 0 \quad (10)$$

$$w_s \frac{\partial c_i}{\partial z} + \frac{\partial}{\partial z} \left(\varepsilon \frac{\partial c_i}{\partial z} \right) = \left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + w \frac{\partial}{\partial z} \right) c_{i-1} \quad \text{for } i \geq 1 \quad (11)$$

respectively where

$$c = \sum_{i=0}^n c_i + 0(\delta^{n+1}) \quad (12)$$

noting that

$$0(c_i) = \delta 0(c_{i-1}) \quad (13)$$

A new dimensionless vertical coordinate is defined by

$$\zeta = \frac{z - (z_a + z_b)}{h} \quad (14)$$

It should be noted that ζ will in general not be independent of x and t .

Let all depth averaged quantities denoted by \bar{u} , \bar{c} etc., be averaged over the depth so that for example

$$\bar{u} = \frac{1}{h} \int_{z_a + z_b}^{z_s} u \, dz = \int_0^1 u \, d\zeta \quad (15)$$

Let the velocity profile be described by

$$u = \bar{u} p(\zeta) \quad \text{where} \quad \bar{p} = 1 \quad (16)$$

Now the basic equations (10) and (11) could be transformed to

$$D[c_0] = 0 \quad (17)$$

$$D[c_i] = \left(\frac{h}{w_s} \frac{\partial}{\partial t} + \frac{\bar{u}h}{w_s} p \frac{\partial}{\partial x} + \frac{w}{w_s} \frac{\partial}{\partial z} \right) c_{i-1} \quad \text{for} \quad i \geq 1 \quad (18)$$

where

$$D[\] = \frac{\partial}{\partial \zeta} + \frac{\partial}{\partial \zeta} \left(\varepsilon' \frac{\partial}{\partial \zeta} \right) \quad (19)$$

$$\varepsilon' = \frac{\varepsilon}{w_s h}$$

For the purpose of constructing the asymptotic series solution it is assumed that only c_0 contributes to the mean concentration, i.e.,

$$\bar{c}_0 = \bar{c}(x, t) \quad (20)$$

$$\bar{c}_i = 0 \quad \text{for all} \quad i \geq 1 \quad (21)$$

Other ways of normalizing the solution seem to be possible, but these have not been considered in detail. Furthermore, as (17) is a differential equation in ζ only, it is possible to separate variables and write the zero-order term as

$$c_0 = \bar{c} \phi_0(\zeta) \quad (22)$$

implying that

$$\bar{\phi}_0 = 1$$

Then (17) will reduce to

$$D[\phi_0] = 0 \quad (23)$$

The surface boundary condition (2) must be satisfied for all values of $n \geq 0$. Thus

$$\phi_0 + \varepsilon' \frac{\partial \phi_0}{\partial \zeta} = 0 \quad \text{at} \quad \zeta = 1 \quad (24)$$

and

$$c_i + \varepsilon' \frac{\partial c_i}{\partial \zeta} = 0 \quad \text{at} \quad \zeta = 1 \quad \text{for} \quad i \geq 1 \quad (25)$$

The function ϕ_0 can be obtained by solving equation (23) with conditions (20) and (24). This leads to the usual equilibrium concentration profile, which is completely determined by the distribution of ε' .

So far, the solution is determined as a function of ζ , but with an unknown coefficient $\bar{c}(x, t)$, which, together with its derivatives, occurs in all terms of the asymptotic series. This is elaborated in the next paragraph. In order to fix the depth-averaged concentration \bar{c} , the bed boundary condition (3) is applied to the full asymptotic solution:

$$c_a = \sum_{i=0}^n c_i(x, z_a, t) \quad (26)$$

4 The case of slowly varying flow

If the flow is so slowly varying that it is possible to neglect the vertical velocity component as well as the x and t derivatives of ϕ_0 , p and ζ , equation (18) will reduce to

$$D[c_i] = \frac{h}{w_s} \frac{\partial c_{i-1}}{\partial t} + \frac{\bar{u}h}{w_s} p(\zeta) \frac{\partial c_{i-1}}{\partial x} \quad i \geq 1 \quad (27)$$

Substituting for c_0 from (22) for $i = 1$,

$$D[c_1] = \frac{h}{w_s} \frac{\partial \bar{c}}{\partial t} \phi_0(\zeta) + \frac{\bar{u}h}{w_s} \frac{\partial \bar{c}}{\partial x} p(\zeta) \phi_0(\zeta)$$

which is a second order differential equation in ζ only, with the boundary conditions (21) and (25). Thus it is possible to write

$$c_1 = a_{21}(\zeta) \frac{h}{w_s} \frac{\partial \bar{c}}{\partial t} + a_{22}(\zeta) \frac{\bar{u}h}{w_s} \frac{\partial \bar{c}}{\partial x} \quad (28)$$

where

$$a_{21}(\zeta) = D^{-1}[\phi_0], \quad a_{22}(\zeta) = D^{-1}[p\phi_0] \quad (29)$$

and $D^{-1}[\]$ is defined by

$$D^{-1}[g] = f \quad \text{if} \quad D[f] = g \quad \text{where} \quad \left[f + \varepsilon' \frac{\partial f}{\partial \zeta} \right]_{\zeta=1} = 0 \quad \text{and} \quad \bar{f} = 0$$

A general analytical expression for f in terms of g and ϕ_0 is given by Galappatti (1983). It is seen that the first-order term c_1 is again expressed in terms of the depth-averaged concentration \bar{c} and (standard) similarity functions a_{21} and a_{22} .

The complete first-order solution is obtained by adding c_0 and c_1 :

$$c = a_{11}(\zeta)\bar{c} + a_{21}(\zeta) \frac{h}{w_s} \frac{\partial \bar{c}}{\partial t} + a_{22}(\zeta) \frac{\bar{u}h}{w_s} \frac{\partial \bar{c}}{\partial x} \quad (30)$$

where $a_{11}(\zeta) = \phi_0(\zeta)$

The first order steady solution would be

$$c = a_{11}(\zeta)\bar{c} + a_{22}(\zeta) \frac{\bar{u}h}{w_s} \frac{\partial \bar{c}}{\partial x} \quad (31)$$

If it is necessary to move to a higher order of approximation it is possible to substitute (28) in (27) for $i=2$ and so on. In general c_i will be expressed in terms of the i -th order derivatives of \bar{c} in x and t .

The second order steady solution can be shown to be

$$c = a_{11}(\zeta)\bar{c} + a_{22}(\zeta) \frac{\bar{u}h}{w_s} \frac{\partial \bar{c}}{\partial x} + a_{33}(\zeta) \frac{\bar{u}h}{w_s} \frac{\partial}{\partial x} \left(\frac{\bar{u}h}{w_s} \frac{\partial \bar{c}}{\partial x} \right) \quad (32)$$

where $a_{33}(\zeta) = D^{-1}(pa_{22})$.

Equations (30), (31) and (32) are expressions for the concentration which satisfy the mass-balance equation (1) and the surface boundary condition subject to the assumptions about orders of magnitude. The vertical profile function $a_{11}(\zeta)$, $a_{21}(\zeta)$ etc. can be determined in advance if the velocity profile, the equilibrium concentration profile ϕ_0 and z_a/a are known.

If the assumption of local near-bed equilibrium is used, the bed boundary condition (3) can be reformulated in terms of the local equilibrium concentration \bar{c}_e as

$$c_a = \bar{c}_e \phi_0(0) = \bar{c}_e \gamma_{11} \quad (33)$$

where $\gamma_{11} = a_{11}(0) = \phi_0(0)$.

Application of the condition (26) then gives for slowly varying flow:

a. First order unsteady solution

$$\gamma_{11}\bar{c}_e = \gamma_{11}\bar{c} + \gamma_{21} \frac{h}{w_s} \frac{\partial \bar{c}}{\partial t} + \gamma_{22} \frac{\bar{u}h}{w_s} \frac{\partial \bar{c}}{\partial x} \quad (34)$$

where $\gamma_{21} = a_{21}(0)$, $\gamma_{22} = a_{22}(0)$

b. First order steady solution

$$\gamma_{11}\bar{c}_e = \gamma_{11}\bar{c} + \gamma_{22} \frac{\bar{u}h}{w_s} \frac{\partial \bar{c}}{\partial x} \quad (35)$$

c. Second order steady solution

$$\gamma_{11}\bar{c}_e = \gamma_{11}\bar{c} + \gamma_{22} \frac{\bar{u}h}{w_s} \frac{\partial \bar{c}}{\partial x} + \gamma_{33} \frac{\bar{u}h}{w_s} \frac{\partial}{\partial x} \left(\frac{\bar{u}h}{w_s} \frac{\partial \bar{c}}{\partial x} \right) \quad (36)$$

where $\gamma_{33} = a_{33}(0)$.

These are all differential equations that describe the variation of \bar{c} along the x -direction, and in the case of (34), with time. The coefficients of these equations can be determined in advance if at every point the velocity profile and the equilibrium concentration profile are known. The boundary conditions for the solution of these equations have to be given in terms of \bar{c} at the upstream boundary and, in the case of unsteady equations, at time zero. The solution of \bar{c} is a one-

dimensional computation of the situation considered is 2-d. Similarly, a 3-d case can be reduced by one dimension.

Once the depth-averaged concentration \bar{c} has been determined, concentration profiles can be computed from (30), (31) or (32). All related quantities, such as total transport in suspension and rate of entrainment near the bed, can also be derived as shown in section 6.

5 The complete first order quasi-steady solution

In applications (e.g., siltation of trenches in par. 9) for non-uniform flow, it may be necessary to include the vertical velocity component or the variation of the shapes of the velocity profiles or ϕ_0 profiles along the x -axis. The analysis given below assumes that bed level changes occur so slowly that the sediment transport process can be considered to be quasi-steady. It should be noted that the short length scales associated with highly non-uniform flows will tend to undermine the validity of the basic assumption that $UH/w_s L$ is small. However, the actual limits of applicability of the present model will have to be determined by experimentation. It is assumed that velocity profiles are logarithmic and the normalised profile $p(\zeta)$ is determined by the single parameter

$$f_* = \kappa \frac{\bar{u}}{u_*} \quad (37)$$

The magnitude of the diffusion coefficient is assumed to be determined by the shear velocity. Using the steady state equation of continuity it can then be shown that (see appendix):

$$\frac{\partial \phi_0}{\partial x} = \frac{\partial \phi_0}{\partial \zeta} \frac{\partial \zeta}{\partial x} + g_2(\zeta) \left[\frac{1}{h} \frac{\partial h}{\partial x} + \frac{1}{(1 - A/f_*)} \frac{1}{f_*} \frac{\partial f_*}{\partial x} \right] \quad (38)$$

where

$$A = \beta \ln \left(\frac{\beta + 1}{\beta} \right)$$

$$\beta = z_a/h$$

$g_2(\zeta)$ = a function that can be expressed in terms of previously defined functions

From the equation of continuity, the vertical velocity component can be shown to be (see appendix):

$$w = r(\zeta) \frac{\bar{u}h}{f_*} \frac{\partial f_*}{\partial x} - p(\zeta) \bar{u}h \frac{\partial \zeta}{\partial x} \quad (39)$$

where

$$r = - \left[\int_{\zeta}^1 p \, d\zeta \right] / (1 - A/f_*) + (1 - \zeta)$$

The first term on the right hand side represents the (small) component of vertical velocity generated by changing relative roughness and the second term the component due to the shift of streamlines.

For steady flow with vertical velocity (18) becomes for $i = 1$

$$D[c_1] = \frac{\bar{u}h}{w_s} p \frac{\partial c_0}{\partial x} + \frac{w}{w_s} \frac{\partial c_0}{\partial z}$$

As $c_0 = \phi_0(\zeta)\bar{c}(x)$ this gives

$$D[c_1] = \frac{\bar{u}h}{w_s} p\phi_0 \frac{\partial \bar{c}}{\partial x} + \bar{c} \left[pg_2 \frac{\bar{u}}{w_s} \frac{\partial h}{\partial x} + \left\{ \frac{pg_2}{(1-A/f_s)} + r \frac{\partial \phi_0}{\partial \zeta} \right\} \frac{\bar{u}h}{f_s w_s} \frac{\partial f_s}{\partial x} \right]$$

This can be integrated using the boundary conditions (21) and (25) to yield

$$c_1 = a_{22}(\zeta) \frac{\bar{u}h}{w_s} \frac{\partial \bar{c}}{\partial x} + \left[e_1(\zeta) \frac{\bar{u}}{w_s} \frac{\partial h}{\partial x} + e_2(\zeta) \frac{\bar{u}h}{w_s} \frac{1}{f_s} \frac{\partial f_s}{\partial x} \right] \quad (40)$$

where

$$e_1 = D^{-1}[pg_2], \quad e_2 = D^{-1} \left[pg_2/(1-A/f_s) + r \frac{\partial \phi_0}{\partial \zeta} \right]$$

Thus the first order solution can be obtained by adding c_0 and c_1 :

$$c = \left[a_{11} + e_1 \frac{\bar{u}}{w_s} \frac{\partial h}{\partial x} + e_2 \frac{\bar{u}h}{w_s} \frac{1}{f_s} \frac{\partial f_s}{\partial x} \right] \bar{c} + a_{22} \frac{\bar{u}h}{w_s} \frac{\partial \bar{c}}{\partial x} \quad (41)$$

Applying the bed boundary condition (3) gives the complete first order solution for steady, non-uniform flow:

$$\gamma_{11}\bar{c}_e = \left[\gamma_{11} + \mu_1 \frac{\bar{u}}{w_s} \frac{\partial h}{\partial x} + \mu_2 \frac{\bar{u}h}{w_s} \frac{1}{f_s} \frac{\partial f_s}{\partial x} \right] \bar{c} + \gamma_{22} \frac{\bar{u}h}{w_s} \frac{\partial \bar{c}}{\partial x} \quad (42)$$

where $\mu_1 = e_1(0)$ and $\mu_2 = e_2(0)$

It is seen that these expressions are the same as those for the slowly-varying case, with the exception of the coefficient of the lowest order term in the mean concentration \bar{c} , which is modified by non-uniformity of the flow.

6 Sediment transport and entrainment

The sediment transport rate is given by

$$S = S_b + S_s \quad (43)$$

where S_b is the bed-load transport rate (if any) and S_s is the suspended sediment transport rate, defined as

$$S_s = \int_{z_b+z_a}^{z_s} uc \, dz = h \int_0^1 uc \, d\zeta = \bar{u}h \int_0^1 cp(\zeta) \, d\zeta \quad (44)$$

By substituting c from (30), (31), (32) this gives for slowly varying flow:

a. First order unsteady

$$S_s = \alpha_{11}\bar{u}h\bar{c} + \bar{u}h \left[\alpha_{21} \frac{h}{w_s} \frac{\partial \bar{c}}{\partial t} + \alpha_{22} \frac{\bar{u}h}{w_s} \frac{\partial \bar{c}}{\partial x} \right] \quad (45)$$

b. First order steady

$$S_s = \alpha_{11}\bar{u}h\bar{c} + \frac{\bar{u}^2 h^2}{w_s} \alpha_{22} \frac{\partial \bar{c}}{\partial x} \quad (46)$$

c. Second order steady

$$S_s = \alpha_{11} \bar{u} h \bar{c} + \frac{\bar{u}^2 h^2}{w_s} \alpha_{22} \frac{\partial \bar{c}}{\partial x} + \frac{\bar{u}^2 h^2}{w_s} \alpha_{33} \frac{\partial}{\partial x} \left(\frac{\bar{u} h}{w_s} \frac{\partial \bar{c}}{\partial x} \right) \quad (47)$$

d. First order steady, with correction for non-uniformity

$$S_s = \bar{u} h \left\{ \left(\alpha_{11} + \lambda_1 \frac{\bar{u}}{w_s} \frac{\partial h}{\partial x} + \lambda_2 \frac{\bar{u} h}{w_s} \cdot \frac{1}{f_s} \frac{\partial f_s}{\partial x} \right) \bar{c} + \alpha_{22} \frac{\bar{u} h}{w_s} \frac{\partial \bar{c}}{\partial x} \right\} \quad (48)$$

where

$$\alpha_{ij} = \int_0^1 a_{ij} p \, d\zeta$$

and

$$\lambda_i = \int_0^1 e_i p \, d\zeta$$

The sediment entrainment rate E can be determined as the vertical sediment flow at $z = z_a$, or alternatively from the overall sediment balance:

$$E = \frac{\partial}{\partial t} (\bar{c} h) + \frac{\partial}{\partial x} S_s \quad (49)$$

in which the previously found expressions for S_s should be introduced. It is noted that E is an $0(\delta)$ quantity.

As an example, for the first-order steady solution the vertical sediment flux at $z = z_a$ turns out to be:

$$E = \alpha_{11} \bar{u} h \frac{\partial \bar{c}}{\partial x} + 0(\delta^2)$$

Expressions for the other cases can be derived in a similar way (Galappatti, 1983).

7 Computation of profile functions and coefficients

The profile functions $a_{21}(\zeta)$, $a_{22}(\zeta)$... were obtained by numerically solving the equations (29) etc. All vertical integrations were carried out on a uniform grid of 200 intervals from $\zeta = 0$ to $\zeta = 1$ using Simpson's rule with error estimation and correction. The function ϕ_0 used in these computations was based on the parabolic-constant distribution of diffusion coefficient suggested by DHL (1980). The function $p(\zeta)$ was based on a logarithmic velocity profile:

$$u = \frac{u_*}{\kappa} \ln \left(\frac{z}{z_0} \right) \quad (50)$$

It could be shown that the shape of ϕ_0 depends on the two parameters w_s/u_* and β . The expressions derived in this paper are based on constant β . The shape of $p(\zeta)$ can be shown to depend on f_s and β (see Galappatti, 1983).

Typical shapes of the profile functions are shown in Fig. 2. The variation of various coefficients with w_s/u_* are shown in Fig. 3 and 4. An indication of the influence of z_a is also given. It does not

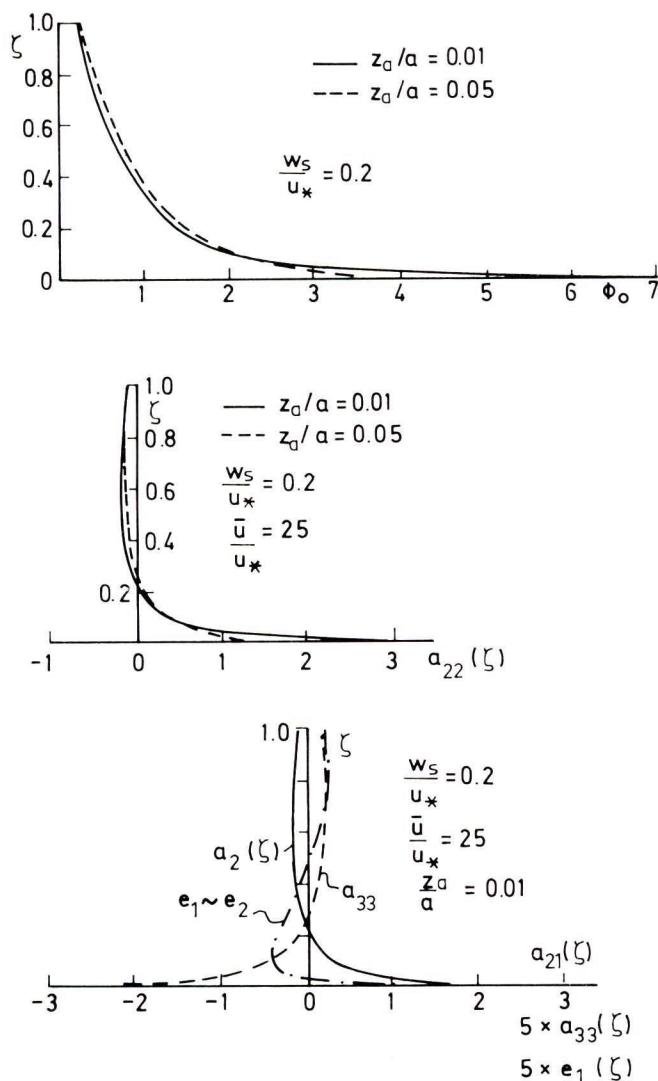


Fig. 2. Typical profile functions.
Fonctions de forme caractéristiques.

affect the profile functions very much, but there is a considerable influence on the γ coefficients, which determine the mean concentration. Therefore, the value of z_a should be selected with care, in combination with the value of c_a to be applied.

The zero-order profile ϕ_0 has about the standard shape as originally derived by Rouse (1937). The first-order correction a_{22} , which is the main contribution of the present theory, is seen to give a steeper profile if $\partial \bar{c} / \partial x > 0$, i.e. roughly speaking, if the concentration is below equilibrium. Conversely, the profile is flattened if $\partial \bar{c} / \partial x < 0$, i.e. in a case of sedimentation. It is obvious that $\partial \bar{c} / \partial x$ should not be too large numerically, as negative concentrations would result. This is related to the basic assumption for the present theory that the length scale should be sufficiently large.

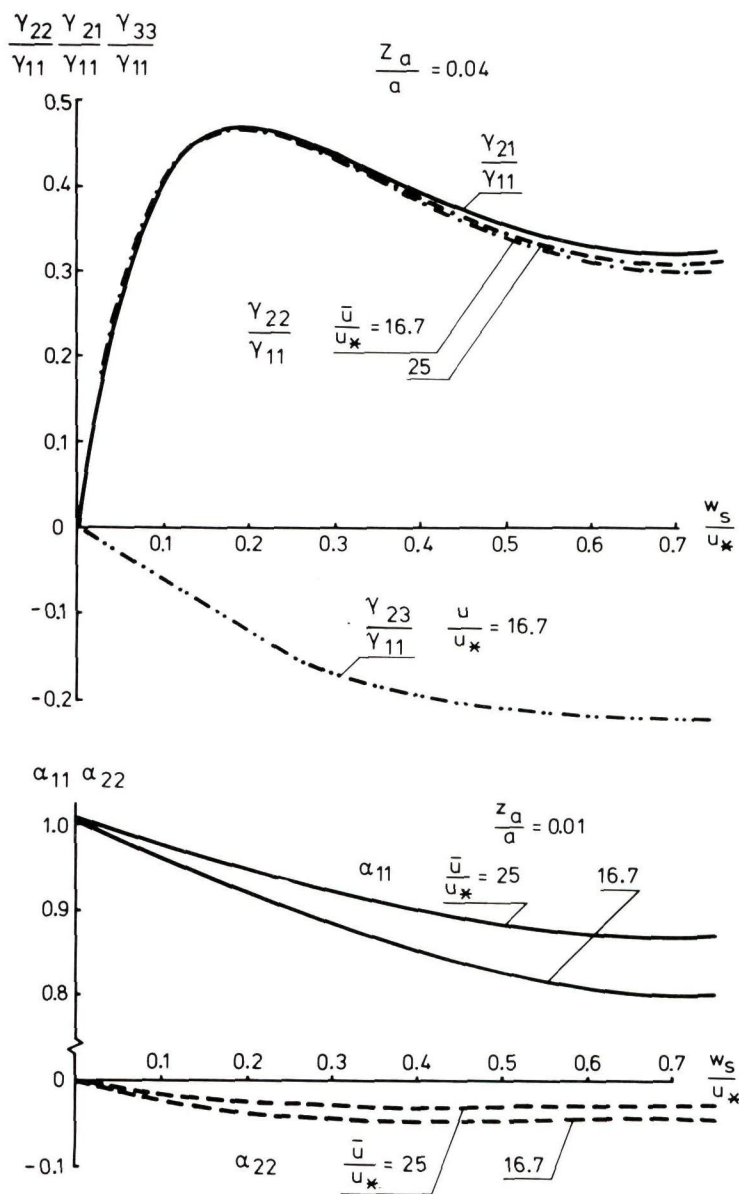


Fig. 3. Behaviour of γ and α coefficients.
Evolution des coefficients γ et α .

8 Adaptation length and time

Consider a slowly varying flow. The governing equation (34) for a first order solution could also be written as

$$\bar{c}_e = \bar{c} + T_A \frac{\partial \bar{c}}{\partial t} + L_A \frac{\partial \bar{c}}{\partial x} \quad (51)$$

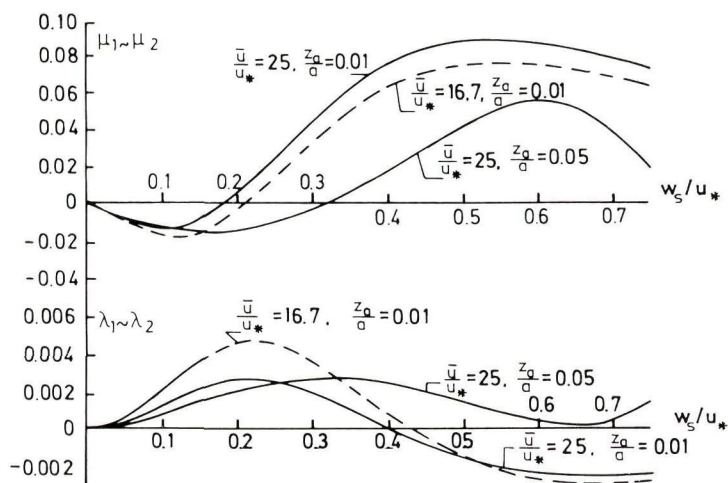


Fig. 4. Behaviour of μ and λ coefficients.
Evolution des coefficients μ et λ .

where the adaptation length and time, L_A and T_A , are given by

$$T_A = \frac{\gamma_{21}}{\gamma_{11}} \frac{h}{w_s} \quad \text{and} \quad L_A = \frac{\gamma_{22}}{\gamma_{11}} \frac{\bar{u}h}{w_s} \quad (52)$$

Equation (51) describes the rate at which the mean concentration approaches the mean equilibrium concentration. In uniform flow where L_A and T_A are constant, $|\bar{c} - \bar{c}_e|$ will decay according to $\exp(-x/L_A)$ or $\exp(-t/T_A)$ along a characteristic. The values of L_A and T_A can be calculated from the local flow parameters and the profiles $\phi_0(\zeta)$ and $p(\zeta)$. They are illustrated in Fig. 5. It is found that the adaptation length is in the order of 20 to 100 times the water depth, roughly, and the adaptation time is the corresponding travel time for a particle moving with the mean velocity. As γ_{22}/γ_{11} does not deviate too much from unity, the scaling assumption $\delta \ll 1$ on which the asymptotic solution is based, means that time and length scales involved in the problem considered, should be large compared with the adaptation time and length. How large is not exactly known and has to be ascertained by comparison with exact solutions and experiments. However, if the scales of the flow are significantly smaller than the adaptation scales, the validity of the original equation (1) becomes questionable, too.

Another way of looking at the validity is noting that the derivative terms in equation (51) should be relatively small by assumption. This evidently implies that $\bar{c}_e - \bar{c}$ should be relatively small, i.e. the theory describes deviations from equilibrium, if not too large. Again, it is unknown how large the deviations may be without violating the assumptions.

In steady uniform flow, the governing equations (35) and (36) take the form

$$\bar{c}_e = \bar{c} + L_1 \frac{\partial \bar{c}}{\partial x} + L_2^2 \frac{\partial^2 \bar{c}}{\partial x^2} + \dots \quad (53)$$

where the coefficients L_1 , L_2 etc. are constant. Thus analytical solutions are possible and these could be compared with the corresponding numerical solution to the full two-dimensional equation. For the purpose of comparison, the upstream ($x=0$) concentration was assumed to be zero

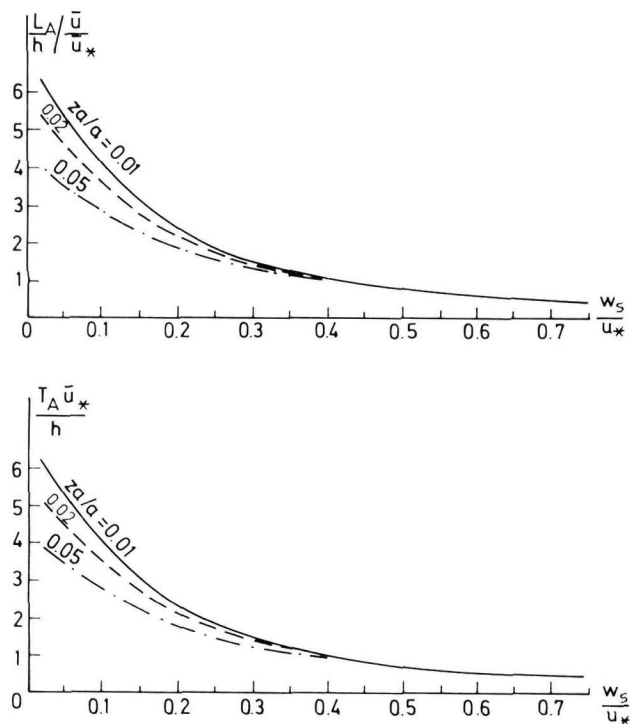


Fig. 5. Dimensionless adaptation length and time, valid for values of the Chezy coefficient between 50 and $75 \text{ m}^{1/2} \text{ s}^{-1}$. The variation due to \bar{u}/u_* is too small to be shown.

Longueur et temps adimensionnels d'adaptation pour des coefficients de Chézy compris entre 50 et 75 (USI). La variation de \bar{u}/u_* est trop faible pour être montrée.

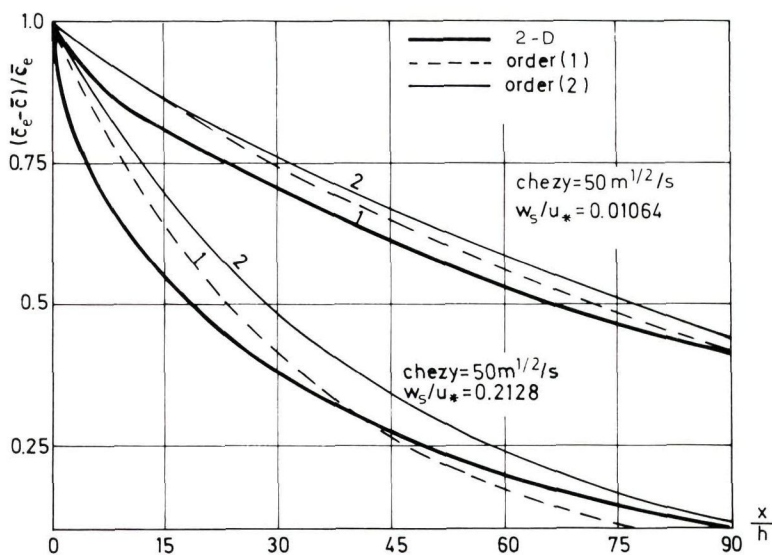


Fig. 6. Adaptation from zero concentration from full 2-d numerical solution and from the present theory. Evolution depuis une concentration nulle dans le cas de la solution bidimensionnelle complète et de la présente théorie.

and the bed boundary condition was defined as

$$[c]_{z=0} = \text{constant}$$

Fig. 6 compares the 1st and 2nd order asymptotic solutions with the full numerical solution given by Flokstra and Vermaas (1984). Initially, a considerable deviation can be seen, which agrees with the criterion that the deviation between \bar{c} and \bar{c}_e should not be too large. In fact, starting the asymptotic solutions at a mean concentration not so far from equilibrium results in a much better agreement.

The values of the coefficients obtained were such that only one positive adaptation length exists even in second (or higher) order solutions. Thus only one upstream boundary condition could be applied. The second order solution models the far downstream adaptation length better while the first order solution does better further upstream.

9 Siltation of a dredged trench

The full first order quasi-steady equation (42) was applied to a flume experiment reported in DHL (1980). The experiment consisted of allowing a uniform flow with a fully developed sediment concentration profile flow over a gentle-sided (1 : 10) trench. The bed consisted of fine sand $D_{50} \sim 160 \mu\text{m}$, the mean flow velocity was 0.51 m/s and the flow depth 0.39 m. The trench was 0.16 m deep initially. Its deformation was measured.

DHL (1980) analysed the experiment using its full 2-d numerical model. A comparison is now made with the asymptotic solution described in this paper. Actually, it is not certain whether the theory is fully valid in this case. The computations based on equation (42) were carried out using as far as possible the same values as in DHL (1980). However, the following exceptions have to be made.

1. The ratio z_a/a was not kept constant in the 2-d computation. An average value was used here.
2. The 2-d computation was carried out using different bed boundary conditions for erosion and deposition. In the asymptotic solution a single type of boundary condition based on the bed concentration was used.

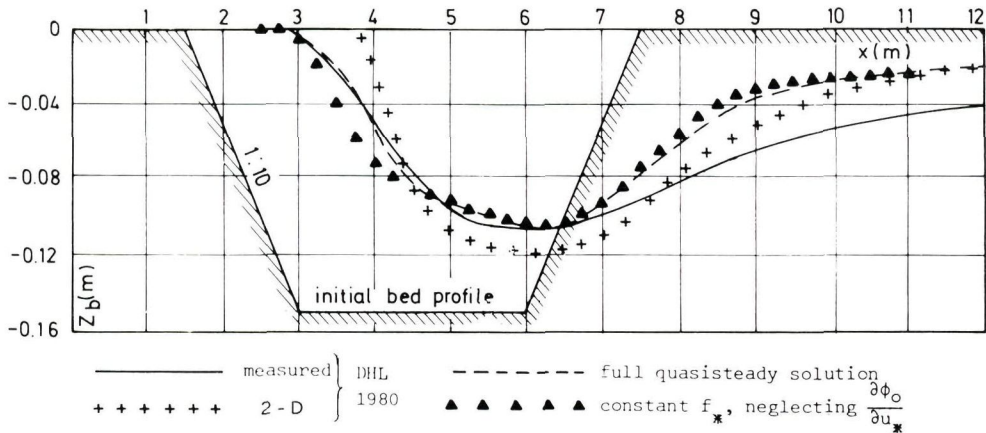


Fig. 7. Siltation of a dredged trench: bed profile at 7,5 hours. Comparison between measurements, full 2-d numerical solution and present theory.

Envasement d'un chenal dragué: profil après 7,5 heures. Comparaison entre les mesures, la solution bidimensionnelle complète et la présente théorie.

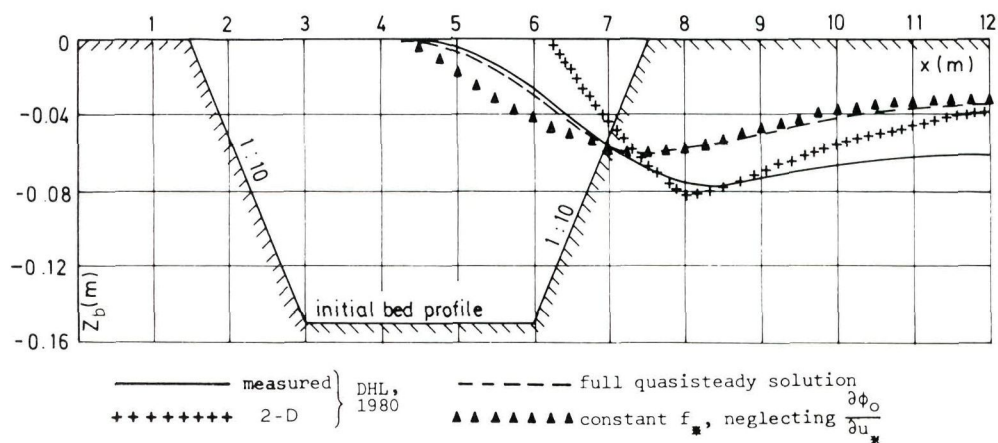


Fig. 8. Same as Fig. 7 at 15 hours.

Idem à la figure 7, mais au bout de 15 heures.

3. The asymptotic model was calibrated so as to match the observed propagation velocity of the bottom disturbance by adjusting the proportion of suspended sediment load to bed load while keeping the total sediment load unchanged.

This calibration is possible by adjusting the value of the bed boundary condition for suspended load and the value of the coefficient in the bed-load transport formula. The adjustment resulted in a bed load $S_b = 0.018$ kg/s/m and a suspended load of $S_c = 0.022$ kg/s/m at the entrance section. As a comparison, the study of DHL (1980) used the figures of 0.010 and 0.030, respectively. The calculated and observed bed profiles after 7.5 hours and 15 hours are shown in Fig. 7 and 8. The calculated values given are from the 2-d computation and the asymptotic solution with and without the $\partial f_s / \partial x$ and $\partial h / \partial x$ terms given in equation (42). The agreement is good especially on the upstream part of the trench. It is expected that the applicability of the theory will be better for more gradually varied flow in large regions.

10 Conclusions

This paper described the first steps taken in developing an alternative approximate solution to the mass-balance equation for suspended sediment. It is felt that this solution will allow the gap between the depth-averaged solutions and the full two-dimensional solutions to be bridged. The asymptotic approach yields equations that can be written in terms of depth-averaged quantities and has the potential for extension to a third dimension.

The present model could not be verified extensively, but a comparison with a full 2-d approach indicates that reasonable results are obtained. An indirect test has been made by reproducing an experiment for the siltation of a trench, with reasonable success.

Notations

- a water depth
- a_{ij} profile function
- c concentration of suspended sediment

\bar{c}	depth averaged concentration
c_a	concentration at reference level z_a
\bar{c}_e	equilibrium mean concentration
e_i	profile function
E	scale for diffusion coefficient; also: entrainment
f_e	friction coefficient
g_i	profile function
h	reduced water depth $a - z_a$
L_A	adaptation length
p	dimensionless velocity profile
S_b	bed load transport
S_s	suspended load transport
t	time
T_A	adaptation time
u, w	velocity component in x, z direction
\bar{u}	depth averaged velocity
u_e	bed shear velocity
w_s	settling velocity
x, z	horizontal and vertical coordinates
z_a	level above bottom where concentration boundary condition is applied
δ	ratio of scales
ε	turbulent diffusion coefficient
ζ	dimensionless vertical coordinate
κ	Von Karman's constant
ϕ_0	normalised equilibrium concentration profile

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APPENDIX

Details for non-uniform flow

Once the horizontal velocity component is given as a function of x , z and t , the vertical velocity component w can be found from the equation of continuity (for steady flow)

$$w = \frac{\partial}{\partial x} \int_z^{z_s} u \, dz \quad (\text{A1})$$

From equation (16), this gives

$$w = \frac{\partial}{\partial x} (\bar{u} h P(\zeta)) \quad (\text{A2})$$

where

$$P(\zeta) = \int_{\zeta}^1 p(\zeta) \, d\zeta$$

If a logarithmic velocity profile is assumed

$$u = \frac{u_*}{\kappa} \ln \frac{z}{z_0}$$

then, by definition

$$\bar{u} h = \int_{z_a}^{z_a+h} u \, dz = \frac{u_*}{\kappa} h \left\{ \beta \ln \frac{\beta+1}{\beta} + \ln(\beta+1)h - \ln z_0 - 1 \right\}$$

Therefore (notation see equation (38)):

$$p(\zeta) = 1 + \frac{1}{f_*} \left\{ \ln \frac{\zeta + \beta}{\beta + 1} - A + 1 \right\} \quad (\text{A3})$$

Introducing this into equation (A2) the result is equation (39).

The equilibrium concentration profile follows from DHL, 1980:

$$\phi_0 = B \exp \{Zf(\zeta)\} \quad (\text{A4})$$

where $f(\zeta)$ is a composite function of ζ and $Z = w_s / \kappa u_*$. Then

$$\frac{\partial \phi_0}{\partial x} = \frac{\partial \phi_0}{\partial z} \frac{\partial \zeta}{\partial x} + \frac{\partial \phi_0}{\partial Z} \frac{\partial Z}{\partial x} \quad (\text{A5})$$

From (A4):

$$\frac{\partial \phi_0}{\partial Z} = f(\zeta) \phi_0(\zeta) + \frac{\partial B}{\partial Z} \exp \{Zf(\zeta)\} \quad (\text{A6})$$

If this is integrated between $\zeta = 0$ and 1, and taking the properties of ϕ_0 into account, we find

$$\frac{\partial B}{\partial Z} = -B \int_0^1 f \phi_0 \, d\zeta$$

so

$$\frac{\partial \phi_0}{\partial Z} = \phi_0(\zeta) \left\{ f(\zeta) - \int_0^1 f \phi_0 \, d\zeta \right\}$$

Further,

$$\frac{\partial Z}{\partial x} = \frac{\partial Z}{\partial(w_s/u_*')} \frac{\partial(w_s/u_*')}{\partial x} = -\frac{w_s}{u_*'} \frac{\partial Z}{\partial(w_s/u_*')} \frac{1}{u_*'} \frac{\partial u_*'}{\partial x} \quad (\text{A7})$$

Finally, from equation (37)

$$\frac{1}{f_*'} \frac{\partial f_*'}{\partial x} = \frac{1}{\bar{u}} \frac{\partial \bar{u}}{\partial x} - \frac{1}{u_*'} \frac{\partial u_*'}{\partial x} \quad (\text{A8})$$

Integration of the equation of continuity gives:

$$\bar{u}h(1 - A/f_*) = \text{constant}$$

so (A8) becomes

$$\frac{1}{u_*'} \frac{\partial u_*'}{\partial x} = -\frac{1}{h} \frac{\partial h}{\partial x} - \frac{1}{(1 - A/f_*)} \frac{1}{f_*'} \frac{\partial f_*'}{\partial x} \quad (\text{A9})$$

Substituting (A6)... (A9) into (A5) yields equation (38) with

$$g_2(\zeta) = \frac{w_s}{u_*'} \frac{\partial \phi_0}{\partial Z} \frac{\partial Z}{\partial(w_s/u_*')} \quad (\text{A10})$$

The details of this function depend on the assumed distribution of the eddy diffusion coefficient ε .