

NOTES

Estimating coefficients in one-dimensional depth-averaged sediment transport model

Qing-Chao Guo and Yee-Chung Jin

Abstract: Various coefficients in sediment transport models must be accounted for. Models based on depth-averaged equations and sediment carrying capacity formula contain some coefficients: α , k , and m . At the present, no widely acceptable method has been developed for determining the values of these coefficients. The focus of this paper is in the development of semi-theoretical formulas for estimating these coefficients such that, in practical applications, the uncertainty involved in selecting coefficients is minimized. Model verification shows that the coefficients obtained from the proposed formulas give a good simulation of the channel bed deformation. In addition, Rouse's equation for sediment concentration distribution will become solvable because the reference concentration can be determined from the derived expression for α . The simulated concentration profiles obtained by solving the Rouse's equation and α formula agree reasonably well with the measured data.

Key words: depth-averaged model, sediment transport, sediment-carrying capacity.

Résumé : Plusieurs coefficients doivent être considérés dans les modèles de transport des sédiments. De façon similaire, les modèles basés sur les équations avec profondeurs moyennes et la formule de capacité de transport contiennent quelques coefficients: α , k et m . Pour le moment, aucune méthode acceptable en général n'a été développée pour la détermination de leurs valeurs. Le but de cet article est le développement de plusieurs formules semi-théoriques afin d'estimer ces coefficients de sorte que, dans les applications pratiques, l'incertitude inhérente à la sélection de ces coefficients est minimisée. La vérification avec un modèle montre que les coefficients obtenus par les formules proposées produisent une bonne simulation de la déformation du lit d'un canal. En plus, l'équation de Rouse pour la distribution de la concentration en sédiments va pouvoir être résolue, car la concentration de référence peut être déterminée par le biais de l'expression pour le coefficient α . Les profils de concentration simulés par la résolution de l'équation de Rouse et la formule pour α concordent raisonnablement avec les données mesurées.

Mots clés : modèle avec profondeurs moyennes, transport de sédiments, capacité de transport de sédiments.

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Introduction

The model using depth-averaged equations and sediment carrying capacity equation (Lin et al. 1983; Guo and Jin 1999) contains three important coefficients: the adjustment coefficient α , coefficient k , and exponent m in the formula for sediment carrying capacity. Theoretically, α is the ratio of the near-bed concentration, s_a , and the cross-sectional av-

erage concentration, s_0 , in the equilibrium state. In reality, the flow-sediment system is usually in a non-equilibrium state. Under non-equilibrium conditions, both the near-bed and the average sediment concentrations are different from those in the equilibrium state. However, for most alluvial rivers with fine sediment, the vertical distributions of suspended sediment concentrations in the two states are not significantly different (Lin et al. 1983). Hence, α can be considered to be approximately the same for both equilibrium and non-equilibrium states and may be evaluated assuming the system is in equilibrium. Lin et al. (1983) presented an expression for calculating α for fine sediment; Han (1980) used field data of natural rivers to calibrate α and proposed the values of 0.25 and 1.0 for deposition and erosion, respectively. It appears that Han's method depends to a large extent on the user's experience. As such, a method is still needed to determine α for the general purpose of sediment transport.

The selection of a formula for the sediment-carrying capacity is another barrier that river engineers encounter. Based on experimental studies and a large quantity of field data, re-

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searchers (Wuhan 1959; Bagnold 1966; Velikanov 1933 (cited by Simons and Senturk 1992)) gave a simple and practical formula for sediment-carrying capacity. However, this formula contains two coefficients, k and m . In this study, a method for estimating the values of k and m is proposed based on the existing formulas and verification with the measured data.

Similarly, a one-dimensional model considers only the depth-averaged variables such as velocity and concentration without taking into account the distributions along the channel depth. To improve this shortcoming in the one-dimensional model, Guo and Jin (1999) assumed linear distributions for velocity and suspended sediment concentration along the flow depth. The assumed concentration distribution is simple, but it gives a general understanding of the concentration profile. To better approximate the distribution, Rouse's equation along with the proposed expression for α is used. Resorting to the expression for α alleviates the difficulty that arises in calculating the reference concentration, s_a , which is used in Rouse's equation.

Coefficients in the model

The accuracy of the model (Guo and Jin 1999) depends strongly on the coefficients α , k , and m . These coefficients are derived in the following sections.

Sediment-carrying capacity coefficient k and exponent m

Sediment-carrying capacity is the ability of the flow to transport sediment without any deposition and erosion. Bagnold (1966) used the concept of suspended-load work to derive a formula for calculating suspended sediment transport rate as follows:

$$[1] \quad q_s = \frac{\gamma_s}{\gamma_s - \gamma} (1 - e_b) e_s \tau_0 \frac{u_0^2}{\omega}$$

where q_s is the suspended sediment discharge expressed as the dry weight per unit time and width; γ and γ_s are the specific weights of clear water and sediment, respectively; e_b and e_s are the bed-load and suspended sediment transport efficiencies, respectively; τ_0 is the bed shear stress; u_0 is the depth-averaged velocity; and ω is the settling velocity.

If q_s is written as $s_* q = s_* h u_0$, and τ_0 is written as $\rho u_*^2 = \rho (\sqrt{g u_0} / C)^2 = \gamma u_0^2 / C^2$, eq. [1] can be rearranged as

$$[2] \quad s_* = \frac{\gamma \gamma_s}{\gamma_s - \gamma} \frac{(1 - e_b) e_s}{C^2} \frac{u_0^3}{h \omega}$$

where s_* is the sediment-carrying capacity; q is the unit width discharge; h is the flow depth; ρ is the water density; u_* is the bed shear velocity; g is the gravitational acceleration; and C is the Chezy coefficient.

Based on laboratory data, Bagnold (1966) gave a value of 0.01 for the coefficient $(1 - e_b) e_s$. For natural rivers, Rubey (1933) suggested that the coefficient $(1 - e_b) e_s$ in eq. [2] is approximately 0.025 instead of 0.01.

After analyzing a large quantity of laboratory data and field data from the Yangtze River, the Yellow River, and reservoirs, Wuhan (1959) reported that there is a strong relationship between the carrying capacity and the comprehensive

factor $u_0^3 / g h \omega$ and gave an expression for the carrying capacity, s_* , as follows:

$$[3] \quad s_* = k' \left(\frac{u_0^3}{g h \omega} \right)^m = k \left(\frac{u_0^3}{h \omega} \right)^m$$

in which k' , k , and m are coefficients. The unit for s_* is kg/m^3 with the depth and velocity expressed in metres and metres per second, respectively.

Comparing eqs. [2] and [3], it was found that the sediment-carrying capacity, s_* , is closely related to the factor $u_0^3 / h \omega$. Equation [2] expresses a linear relationship between s_* and the factor $u_0^3 / h \omega$, whereas eq. [3] gives an exponential relationship. In addition, Bagnold's equation (eq. [2]) can be considered as a special case ($m = 1.0$) of formula [3]. Thus, from eqs. [2] and [3], the coefficient k can be estimated as follows:

$$[4] \quad k = \frac{\gamma \gamma_s}{\gamma_s - \gamma} \frac{(1 - e_b) e_s}{C^2}$$

Hence, the coefficient m may be estimated from eq. [3] if the equilibrium concentration s_* is known. Slight variations in the values of k and m estimated from eqs. [3] and [4] are acceptable, since these values are usually obtained under the assumption that the flow-sediment system is in an equilibrium state.

Adjustment coefficient α

Determining the depth-averaged concentration

The suspended sediment distribution equation derived by Rouse (1937) is given by

$$[5] \quad \frac{s}{s_a} = \left[\frac{(1/\eta) - 1}{(1/\eta_a) - 1} \right]^{z_1}$$

where η is the relative flow depth; η_a is the reference relative depth; s is the concentration corresponding to depth η ; s_a is the reference concentration corresponding to η_a ; and $z_1 = \omega/\beta \kappa u_*$ is an exponent.

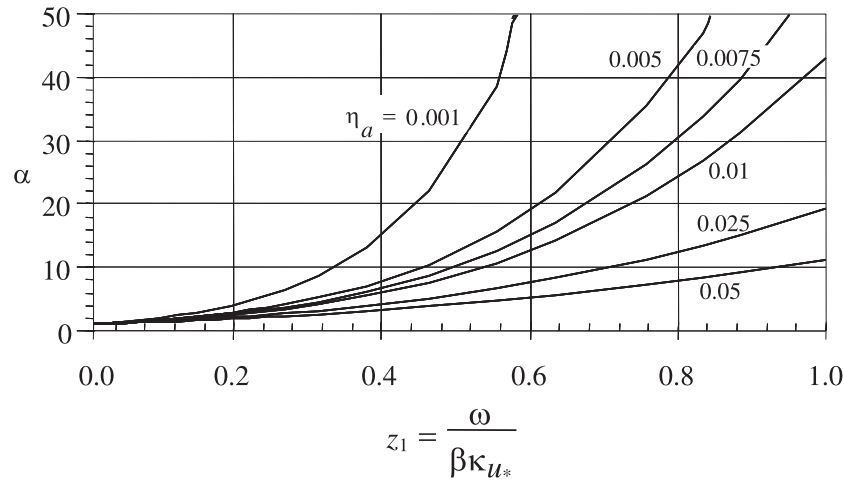
The depth-averaged concentration, s_0 , can be calculated from the following expression:

$$[6] \quad s_0 = \frac{\int_{\eta_a}^1 s u \, d\eta}{u_0}$$

According to Prandtl's mixing length theory (Simons and Senturk 1992), the velocity distribution, u , is given by

$$[7] \quad u = u_0 \left[1 + \frac{\sqrt{g}}{\kappa C} (\ln \eta + 1) \right]$$

where κ is the Von Karman constant. Substituting eqs. [5] and [7] into eq. [6] results in

Fig. 1. Relationship between α and $z_1 = \omega/\beta\kappa u_*$.

$$[8] \quad s_0 = s_a \left(\frac{1}{\eta_a} - 1 \right)^{-z_1} \left[\left(1 + \frac{\sqrt{g}}{\kappa C} \right) \int_{\eta_a}^1 \left(\frac{1}{\eta} - 1 \right) d\eta + \frac{\sqrt{g}}{\kappa C} \int_{\eta_a}^1 \left(\frac{1}{\eta} - 1 \right) \ln \eta d\eta \right]$$

$$[12] \quad \beta = 1 + 2 \left(\frac{\omega}{u_*} \right)^2 \quad \text{for } 0.1 < \frac{\omega}{u_*} < 0.707$$

Determining the coefficient α

By definition, α is the ratio of s_a and s_0 . From eq. [8], α can be expressed as follows:

$$[9] \quad \alpha = \frac{s_a}{s_0} = \left(\frac{1}{\eta_a} - 1 \right)^{z_1} \left[\left(1 + \frac{\sqrt{g}}{\kappa C} \right) \int_{\eta_a}^1 \left(\frac{1}{\eta} - 1 \right) d\eta + \frac{\sqrt{g}}{\kappa C} \int_{\eta_a}^1 \left(\frac{1}{\eta} - 1 \right) \ln \eta d\eta \right]^{-1}$$

Since the exponent z_1 is expressed in terms of an unknown parameter β , an additional equation is necessary for solving α . van Rijn (1984) defined β as the difference of diffusions between a discrete sediment particle and a fluid particle and gave the following expression for β :

$$[10] \quad \beta = 1 + 2 \left(\frac{\omega}{u_*} \right)^2 \quad \text{for } 0.1 < \frac{\omega}{u_*} < 1.0$$

Substituting β into the expression for z_1 yields

$$[11] \quad z_1 = \frac{\omega}{\beta\kappa u_*} = \frac{1}{\kappa} \left(\frac{\omega}{u_*} \right) \left[1 + 2 \left(\frac{\omega}{u_*} \right)^2 \right]^{-1}$$

For different values of ω/u_* (say 3/5 and 5/6), eq. [5] gives the same concentration distribution, resulting in an erroneous result. To make eq. [11] a one-to-one function, ω/u_* has a value in the range $0.1 < \omega/u_* < 0.707$, instead of $0.1 < \omega/u_* < 1.0$. Therefore, van Rijn's (1984) β -factor equation is rewritten as

Figure 1 shows a group of curves representing the relationship between α and $\omega/\beta\kappa u_*$ for the case of Chezy coefficient $C = 36.0$. The curves are obtained through eq. [9]. It can be seen that α increases with increasing $\omega/\beta\kappa u_*$ values. This is consistent with the fact that the sediment distribution becomes more non-uniform along the depth as the value of $\omega/\beta\kappa u_*$ increases. In contrast, when the value of $\omega/\beta\kappa u_*$ remains constant, α decreases quickly with an increase in reference depth, η_a . This is in agreement with the general observations that sediment concentrations vary suddenly within a small depth from the bottom. An appropriate method for determining a reference depth has not been found. With a lack of field data and in estimating the α value, a reference relative depth, h_a , of 0.005–0.01 is suggested. Upon further investigation, it has been found that for fine sediments, the value of α computed from eq. [9] is consistent with that from Lin et al. (1983). For natural rivers, the parameter $\omega/\beta\kappa u_*$ is usually very small and the value of α from eq. [9] is also very close to the value proposed by Han (1980). These results indicate that the new expression, eq. [9], effectively widens the application extent for predicting α values.

Model verification

The verification experiment was carried out at Delft Hydraulics Laboratory (Galappatti and Vreugdenhil 1985). A flume of length = 30 m, depth = 0.7 m, and width = 0.5 m was used. The suspended sediment concentration was about 0.15 kg/m³. The entrance flow was uniform with a fully developed sediment concentration profile and a depth of 0.39 m and a mean velocity of 0.51 m/s. The fall velocity of a representative particle was 0.013 m/s.

For the given flow conditions and sediment composition, the values of C , u_* , β , and z_1 are calculated to be 40.6, 0.04, 1.21, and 0.67, respectively. Applying eq. [9] and selecting a reference depth, η_a , of 0.0075, the value of α is calculated to be 18.6. The coefficient k was found to be 0.0097 from

Fig. 2. Simulated bed variations using the estimated and modified values.

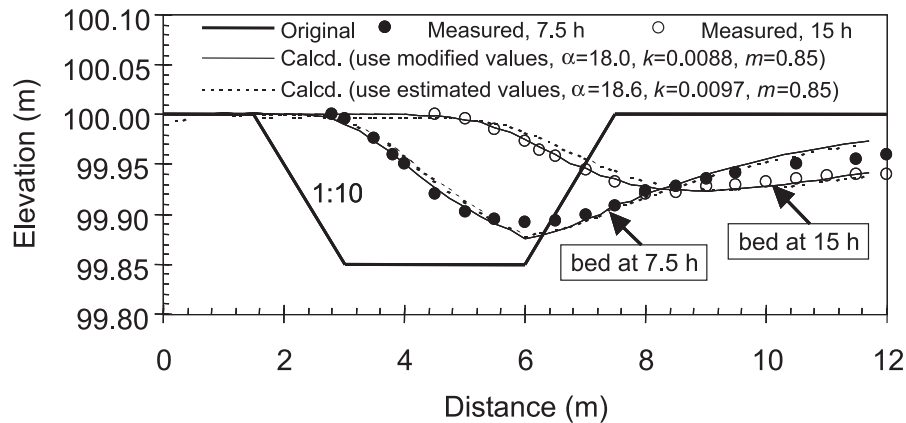
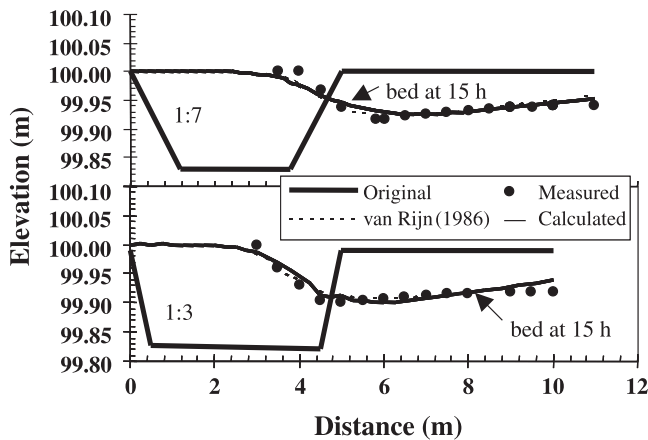


Fig. 3. Comparison of the calculated and measured results at 15 h for the trenches with slopes 1:7 and 1:3.

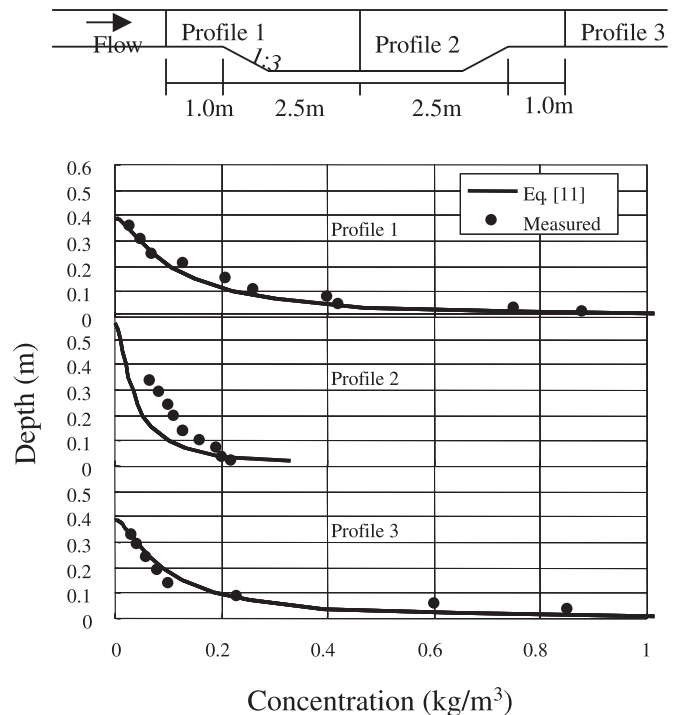


eq. [4]. The coefficient m has a value of 0.84, which is obtained from eq. [3] for an equilibrium concentration of approximately 0.15 kg/m^3 . Using these values in the Guo and Jin (1999) model, the simulated bed variation (dashed line) is plotted against the measured data as shown in Fig. 2. The numerical results agree reasonably well with the experimental data. At 15.0 h run time, the calculated bed variation differs slightly from the experimental results. To obtain a better simulation, the values of the coefficients α , k , and m used are 18.0, 0.0088, and 0.85, respectively. The computed bed elevation (solid line) using these modified values is also plotted in Fig. 2. In both cases, the results indicate a very good agreement at 7.5 and 15 h. This means that the model can simulate not only the final bed variation but also the process of the bed deformation.

Figure 3 shows the simulated bed variations for the trenches with slopes 1:7 and 1:3, respectively. The simulated results also agree well with the measured data. In addition, the simulated bed forms at 15 h derived from van Rijn's two-dimensional model (van Rijn 1986) are also plotted in Fig. 3. Simulation results obtained from the one-dimensional model of this study exhibited the same accuracy as those from two-dimensional models.

As a result of the establishment of the formula for predicting the coefficient α , the reference concentration s_a can be

Fig. 4. Distribution of suspended sediment concentrations.



calculated and therefore Rouse's equation (eq. [5]) becomes solvable. The profiles of sediment concentration from Rouse's equation are presented in Fig. 4. It can be seen that the measured concentration distribution matches quite well the distribution derived from Rouse's equation. This similarity expresses the validity of the method presented in this paper for estimating the coefficients α , k , and m .

Conclusions

Semi-theoretical formulas for estimating the coefficients in the model of depth-averaged equations and the sediment-carrying capacity equation are derived. These formulas provide a guideline for selecting the values of the coefficients.

Verification shows that the coefficients estimated from the formulas give a good simulation of the channel bed defor-

mation. In addition, the assumed linear concentration profiles agree reasonably well with the measured data.

From the established formula for predicting α , the reference concentration s_a in Rouse's equation can be calculated and therefore Rouse's equation can be solved. Comparing the measured and Rouse's concentration profiles indicated a good match.

Because of the complexity of sediment transport, it is difficult to obtain accurate coefficient values. Hence, some modification in the values obtained from formulas is acceptable. In addition, using the measured data to calibrate the three coefficients is always encouraged.

Acknowledgments

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List of symbols

- C Chezy coefficient
- e_b, e_s bed-load and suspended sediment transport efficiencies, respectively
- g gravitational acceleration (m/s^2)
- h water depth (m)
- k sediment-carrying capacity coefficient
- k' coefficient
- m sediment-carrying capacity constant
- q unit wide flow discharge
- q_s suspended sediment discharge expressed as dry weight per unit time and width
- s sediment concentration (kg/m^3)
- s_0 depth-averaged suspended sediment concentration (kg/m^3)
- s_1 half of the concentration difference near the bed and at the surface (kg/m^3)
- s_* sediment-carrying capacity (kg/m^3)
- s_a sediment concentration at the reference depth (kg/m^3)
- u flow velocity in the longitudinal direction (m/s)
- u_0 depth-averaged velocity (m/s)
- u_1 longitudinal velocity in excess of depth-averaged velocity at water surface (m/s)
- u_* shear-stress velocity (m/s)
- α sediment coefficient
- β coefficient describing the difference of diffusion between sediment particle and fluid particle
- γ specific weight of water (kg/m^3)
- γ_s specific weight of sediment (kg/m^3)
- η relative depth, $\eta = (z - z_b)/h$
- η_a reference relative depth
- κ Von Karman constant
- ρ water density (kg/m^3)
- τ_0 bed shear stress (N/m^2)
- ω sediment settling velocity (m/s)