

MODELING SEDIMENT TRANSPORT USING DEPTH-AVERAGED AND MOMENT EQUATIONS

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ABSTRACT: A 1D mathematical model to calculate bed variations in alluvial channels is presented. The model is based on the depth-averaged and moment equations for unsteady flow and sediment transport in open channels. Particularly, the moment equation for suspended sediment transport is originally derived by the assumption of a simple vertical distribution for suspended sediment concentration. By introducing sediment-carrying capacity, suspended sediment concentration can be solved directly from sediment transport and its moment equations. Differential equations are then solved by using the control-volume formulation, which has been proven to have good convergence. Numerical experiments are performed to test the sensitivity of the calibrated coefficients α and k in the modeling of the bed deposition and erosion. Finally, the computed results are compared with available experimental data obtained in laboratory flumes. Comparisons of this model with HEC-6 and other numerical models are also presented. Good agreement is found in the comparisons.

INTRODUCTION

Sediment transport in flow determines the evolution of riverbeds, estuaries, and coastlines. Sediment transport is important because it affects both the functioning and the life span of many hydraulic structures. For example, when a dam is constructed on an alluvial river, sedimentation causes the storage of the reservoir to decrease. In turn, this may affect the benefits of flood control, hydropower generation, and navigation. The clear water released from the reservoir will tend to erode the channel bed downstream until a new equilibrium is established. Severe bed erosion may undermine costly hydraulic projects such as bank-protection works, bridge piers, and diversion structures.

Laboratory experiments to predict sediment transport are generally very time-consuming, costly and, for many practical problems, impossible. Hence, there is a need for mathematical models capable of predicting sediment transport. In recent decades, many numerical models for sediment transport and channel bed variation have been developed. One-dimensional models include those developed by Witkowska (1974), Thomas and Prasuhn (1977), the Hydrologic Engineering Center (HEC) of the U.S. Army Corp of Engineers (*Users'* 1977), Ponce et al. (1979), Han and He (1987), Park and Jain (1987), and Bhallamudi and Chaudhry (1991). Two-dimensional models have been developed by Lin and Shen (1984), Rijn (1986), Celik and Rodi (1988), Shimizu and Itakura (1989), and Rijn et al. (1990).

Each type of model has associated problems. As was pointed out by Dawdy and Vanoni (1986), "none of the movable-bed models evaluated was found to yield wholly satisfactory results." Most 1D models need to choose a certain sediment transport function for a specific study. Unfortunately, the sediment transport function themselves give poor performance (Vanoni 1984). Also, some models use the concept of equilibrium to simulate sediment transport; this may not be appropriate for real-world engineering. Two-dimensional models are obviously computation intensive, and hence, very time-consuming. More importantly, 2D models encounter the problem

of determining the reference concentration of suspended sediment near the bed and the problem of simulating the term for sediment diffusion due to turbulent motion. Although a number of researchers including Rouse (1938), Meyer Peter and Muller (1948), Einstein (1950), Rijn (1984a,b), and Zyserman and Fredsoe (1994) have investigated the bed concentration of suspended sediment, it remains difficult to accurately determine the reference concentration. This is because of the complexity of the mechanism of sediment transport near the bed and the difficulty in selecting a reference level above the bed. After reviewing several methods of determining the reference level and concentration, Celik and Rodi (1988) pointed out that "none of these formulas has been found to be of universal validity." Due to these difficulties, the applications of some models are limited.

The purpose of this paper is to establish a model to simulate bed variations. Bed variation is determined by the difference of the quantities of sediment at the inlet and the outlet. It can be calculated as long as the average concentrations of sediment are known. Thus, it is not necessary to know the vertical profile of sediment concentration. In other words, for most cases of bed variations, a 1D simulation is appropriate.

The present model is a 1D model based on the depth-averaged and moment equations. By using the integrated sediment equation and establishing the relationship between sediment diffusion and sediment-carrying capacity, both the difficulty of determining the reference concentration and the problem of selecting the sediment transport function are avoided. This study attempts to reduce empiricism for the application of sediment transport modeling. At this stage, the model is restricted to the evolution of alluvial rivers with suspended sediment transport. The term "suspended sediment" here means that the sediment moves with flow. Suspended sediment may become "bed load" when deposition occurs. Contrarily, bed load will become suspended sediment if erosion occurs. However, suspended sediment does not include the sediment that moves on the surface of the bed. The governing mean flow and sediment transport equations are described in the next section. Thereafter, the sediment transport model is presented. Also presented is an interesting numerical calculation to illustrate both the adjustments of bed elevation and the sensitivity of coefficients. Finally, comparisons with a number of experimental data are given.

MODEL DEVELOPMENT

Governing Equations

Mean-Flow Equations

Using the depth-averaged flow equations and moment of momentum equation (Steffler and Jin 1993), the 1D partial

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differential equations for unsteady flow with hydrostatic pressure distribution in a rectangular channel (Fig. 1) can be written as

$$\frac{\partial h}{\partial t} + \frac{\partial hu_0}{\partial x} = 0 \quad (1)$$

$$\frac{\partial hu_0}{\partial t} + \frac{\partial hu_0 u_0}{\partial x} = -\frac{1}{3} \frac{\partial hu_1^2}{\partial x} - gh \frac{\partial(h + z_b)}{\partial x} - \frac{\tau_b}{\rho} \quad (2)$$

$$\frac{\partial hu_1}{\partial t} + \frac{\partial hu_0 u_1}{\partial x} = -hu_1 \frac{\partial u_0}{\partial x} + \frac{3}{\rho} (\tau_b - 2\bar{\tau}) \quad (3)$$

where h = flow depth; t = time; u_0 = depth-averaged velocity; x = distance along the channel; u_1 = longitudinal velocity in excess of u_0 at the water surface (Fig. 2); g = gravitational acceleration; z_b = bed elevation; τ_b = bed shear stress; ρ = water density; and $\bar{\tau}$ = depth-averaged shear stress.

Suspended Sediment Transport Equation

Suspended sediment concentrations can be calculated from the mass-balance equation, in which the longitudinal diffusive transport is neglected, as follows (Rijn 1986):

$$\frac{\partial s}{\partial t} + \frac{\partial us}{\partial x} + \frac{\partial ws}{\partial z} = \frac{\partial \omega s}{\partial z} + \frac{\partial}{\partial z} \left(\epsilon_z \frac{\partial s}{\partial z} \right) \quad (4)$$

where s = suspended sediment concentration; u = longitudinal velocity; z = vertical distance from bed; w = vertical velocity; ω = settling velocity; and ϵ_z = dispersion coefficient in z -direction. The depth-averaged sediment transport equation can be derived by integrating (4) over the depth of flow with the use of Leibnitz's rule and the flow boundary conditions (Steffler and Jin 1993) and sediment boundary condition $(\omega s)_{z_b+h} + \epsilon_z(\partial s/\partial z)_{z_b+h} = 0$, resulting in

$$\frac{\partial h\bar{s}}{\partial t} + \frac{\partial h\bar{us}}{\partial x} = -(\omega s)_b - \left(\epsilon_z \frac{\partial s}{\partial z} \right)_b \quad (5)$$

where the subscript b denotes the variable evaluated at the channel bed and overbar for a depth-averaged value. Similar to the analysis for developing the moment of momentum equation by Steffler and Jin (1993), (4) is multiplied by a special weight function, $F = 2(z - \bar{z})/h$ (where $\bar{z} = z_b + h/2$), and is integrated along the water depth. A new equation, i.e., moment equation for sediment, is obtained

$$\begin{aligned} \frac{\partial h\bar{z}\bar{s}}{\partial t} + \frac{\partial h\bar{z}\bar{us}}{\partial x} - h\bar{w}\bar{s} - \bar{z} \frac{\partial h\bar{s}}{\partial t} - \bar{z} \frac{\partial h\bar{us}}{\partial x} &= -h\bar{\omega}\bar{s} - h\epsilon_z \frac{\partial \bar{s}}{\partial z} \\ &+ \frac{h}{2} \omega_b s_b + \frac{h}{2} \left(\epsilon_z \frac{\partial s}{\partial z} \right)_b \end{aligned} \quad (6)$$

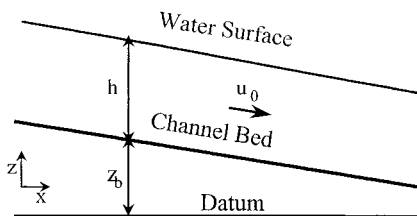


FIG. 1. Definition Sketch

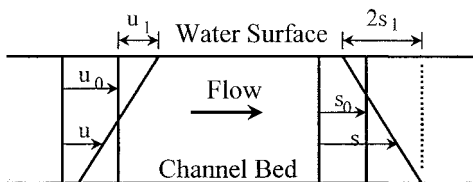


FIG. 2. Definition Sketch for Distributions of u and s

To calculate the variables with overbars (depth-averaged terms), appearing in (5) and (6), the distributions of the velocity and suspended sediment concentration along the water depth are assumed as in Fig. 2

$$u = u_0 + u_1(2\eta - 1) \quad (7)$$

$$s = s_0 + (1 + 2\eta + a)s_1 \quad (8)$$

in which $\eta = (z - z_b)/h$; a = coefficient to be determined; s_1 = half of the concentration difference near the bed and at the water surface; and s_0 = average suspended sediment concentration that will satisfy (Simons and Senturk 1977)

$$hu_0 s_0 = \int_{z_b}^{z_b+h} us \, dz \quad (9)$$

Substituting (7) and (8) into (9), the coefficient a is determined to be $u_1/(3u_0)$.

With (7) and (8), the overbar terms in (5) and (6) become

$$\bar{s} = \frac{1}{h} \int_{z_b}^{z_b+h} s \, dz = s_0 + as_1 \quad (10a)$$

$$\bar{us} = \frac{1}{h} \int_{z_b}^{z_b+h} us \, dz = u_0 s_0 \quad (10b)$$

$$\bar{ws} = \frac{1}{h} \int_{z_b}^{z_b+h} ws \, dz = \bar{w}s_0 + a\bar{w}s_1 - \frac{1}{6}(w_H - w_b)s_1 \quad (10c)$$

$$\bar{zs} = \frac{1}{h} \int_{z_b}^{z_b+h} zs \, dz = \bar{z}s_0 + a\bar{z}s_1 - \frac{1}{6}hs_1 \quad (10d)$$

$$\begin{aligned} \bar{zus} &= \frac{1}{h} \int_{z_b}^{z_b+h} zus \, dz = \bar{z}u_0 s_0 + a\bar{z}u_0 s_1 + \frac{1}{6}ahu_1 s_1 \\ &- \frac{1}{6}h(u_0 s_1 - u_1 s_0) - \frac{1}{3}\bar{z}u_1 s_1 \end{aligned} \quad (10e)$$

where $a = u_1/(3u_0)$; w_H and w_b = vertical velocities at the surface and bed, respectively; and H = water surface level.

To deal with the diffusion terms in (5) and (6), the concept of a sediment-carrying capacity is introduced. If the equilibrium condition is reached, there is no deposition or erosion of the bed and the sediment-carrying capacity s_{b*} near the bed is equal to sediment concentration s_b . The sediment quantity dispersing upward due to turbulent motion should be equal to the downward quantity due to gravitational settling

$$\left(\epsilon_z \frac{\partial s}{\partial z} \right)_b = -\omega_b s_{b*} \quad (11)$$

Further assume that the relationship (11) is appropriate in nonequilibrium and that $s_{b*} = \alpha_1 s_*$ (s_* is the sediment-carrying capacity of the cross section). Eq. (11) then becomes

$$\left(\epsilon_z \frac{\partial s}{\partial z} \right)_b = -\alpha_1 \omega_b s_* \quad (12)$$

In addition, assume that the concentration near the bed s_b also has a relationship with s_0 (the average concentration of the cross section), say $s_b = \alpha_2 s_0$, then the term $\omega_b s_b$ in (5) and (6) can be written as

$$\omega_b s_b = \alpha_2 \omega_b s_0 \quad (13)$$

Bechteler and Schrimpf (1988) considered it sufficient to use the integral mean value of ϵ_z for the calculation of practical sediment transport problems, and also from (8) we get $\partial s/\partial z = -(2s_1/h)$. Thus, the term $h\epsilon_z(\partial s/\partial z)$ in (6) can be written as

$$h\epsilon_z \frac{\partial s}{\partial z} = h\bar{\epsilon}_z \left(-\frac{2s_1}{h} \right) = -2\bar{\epsilon}_z s_1 \quad (14)$$

If the mean settling velocity ω is used to replace the settling velocity near the bed ω_b , and the coefficient α is used to replace α_1 and α_2 , substituting (10), (12)–(14) into (5) and (6) results in

$$\frac{\partial s_0}{\partial t} + \frac{\partial hu_0 s_0}{\partial x} = -\frac{\partial has_1}{\partial t} - \alpha\omega(s_0 - s_*) \quad (15)$$

$$\begin{aligned} \frac{\partial hs_1}{\partial t} + \frac{\partial hu_0 s_1}{\partial x} &= hu_1 \frac{\partial(s_0 + as_1)}{\partial x} + 6\omega(s_0 + as_1) \\ &- 3\alpha\omega(s_0 - s_*) - \frac{12}{h} \varepsilon_z s_1 \end{aligned} \quad (16)$$

Eqs. (15) and (16) are the sediment transport equation and its moment equation for the 1D case, respectively. It should be noted that α_1 will not be equal to α_2 in the nonequilibrium state. In this condition α becomes a comprehensive coefficient.

Sediment Continuity Equation

The sediment continuity equation is also called the bed variation equation. According to continuity of sediment transport, the quantity of bed variation within Δx and Δt should be equal to the quantity of the difference between two cross-sectional sediment transports and the quantity of the difference of concentrations in the water body within Δt . Thus, the bed variation equation can be written as

$$\gamma' \frac{\partial z_b}{\partial t} = -\left(\frac{\partial hs_0}{\partial t} + \frac{\partial hu_0 s_0}{\partial x} + \frac{\partial has_1}{\partial t}\right) \quad (17)$$

where γ' = dry specific weight of bed materials.

This equation is slightly different from the classical equation (Rijn 1986; Park and Jain 1987), because of the distribution assumption for suspended sediment concentration.

Referring to (15), (17) can be further written as

$$\gamma' \frac{\partial z_b}{\partial t} = -\left(\frac{\partial hs_0}{\partial t} + \frac{\partial hu_0 s_0}{\partial x} + \frac{\partial has_1}{\partial t}\right) = \alpha\omega(s_0 - s_*) \quad (18)$$

Assumptions

Bed Shear Stress τ_b

The local bed shear stress τ_b can be computed through the Chézy equation and Manning formula. The expression of τ_b using the Chézy coefficient C or Manning roughness n is as follows:

$$\frac{\tau_b}{\rho} = \frac{gn^2 u_0^2}{h^{1/3}} = \frac{gu_0^2}{C^2} \quad (19)$$

Depth-Averaged Shear Stress $\bar{\tau}$

According to the definition of shear stress, $\bar{\tau}$ can be approximately expressed as

$$\frac{\bar{\tau}}{\rho} \approx \bar{v}_t \frac{\partial u}{\partial z} \quad (20)$$

where \bar{v}_t = depth-averaged eddy viscosity.

Using the mixing length hypothesis recommended by Prandtl

$$\left(\bar{v}_t = l^2 \left| \frac{\partial u}{\partial z} \right|, \quad l = \lambda h\right)$$

and calculating $\partial u / \partial z$ from (7), (20) can be rewritten as

$$\frac{\bar{\tau}}{\rho} \approx 4\lambda^2 h^2 |u_1| u_1 \quad (21)$$

where λ = mixing length coefficient.

Dispersion Coefficient ε_z

The dispersion coefficient for suspended sediment ε_z has a very close relationship with the turbulent diffusion. Many relations for ε_z are derived from either theoretical or experimental investigations. Generally, parabolic profiles are applied (Simons and Senturk 1977). Rijn (1986) used a parabolic-constant profile to calculate sediment distribution. Bechteler and Schrimpf (1988) found that the vertical distribution of ε_z has no significant influence on the settling rates. It was suggested that it should be sufficient to use the integral mean of ε_z for the calculation of many practical sediment transport problems. Thus, ε_z is simply given the average value along the vertical

$$\varepsilon_z = \frac{\kappa}{6} u_* h \quad (22)$$

where κ = von Kármán constant, and $\kappa = 0.4$; and u_* = shear-stress velocity.

Sediment-Carrying Capacity s_*

For a given sediment composition, a certain flow can only carry a certain quantity of sediment without net deposition and erosion. This quantity is called the sediment-carrying capacity and the corresponding flow is said to be under saturation conditions. When the quantity of sediment supply is greater than the capacity, net deposition will occur. This leads to a decrease in sediment concentration until the carrying capacity is reached. Conversely, if the sediment supply is less than the capacity and the riverbed is movable, net scour may occur. The sediment concentration will then increase until the carrying capacity is reached again.

Sediment-carrying capacity should be a function related to hydraulic factors (such as velocity and depth) and sediment factors (such as sediment size and specific weight). A representative formula (Bagnold 1973) for the sediment-carrying capacity can be written as

$$s_* = k \left(\frac{u_0^3}{h\omega} \right)^m \quad (23)$$

where k and m = coefficients (s_* in kg/m^3 , h in m , u in m/s , and ω in m/s). Generally, m is given a value of 0.92 (Han and He 1987).

Numerical Scheme

General Equation

Although the model is 1D, solving the set of differential equations is still complex. To solve the differential equations, a more general form for equations (1)–(3), (15), and (16) is expressed as

$$\frac{\partial h\varphi}{\partial t} + \frac{\partial hu_0 \varphi}{\partial x} = S_\varphi \quad (24)$$

where φ = general variable. When φ is given by 1, u_0 , u_1 , s_0 , and s_1 , (1)–(3), (15), and (16) can be obtained. S_φ is the source term. For the sake of clarity, Table 1 is given to explain (24).

Computation Grid

Solving the set of equations is now transferred to solving the general equation (24). Control-volume formulation (Patanar 1980) is used to discretize the differential equation [(24)]. This formulation has merit because the resulting solution would imply that the integral conservation of quantities such as mass and momentum is exactly satisfied over a group of

TABLE 1. Explanation of Eq. (24)

Equation (1)	ϕ (2)	S_ϕ (3)
Continuity for water (1)	1	0
Momentum for water (2)	u_0	$-\frac{1}{3} \frac{\partial hu_1^2}{\partial x} - gh \frac{\partial(h+z_b)}{\partial x} - \frac{\tau_b}{\rho}$
Moment for water (3)	u_1	$-hu_1 \frac{\partial u_0}{\partial x} + \frac{3}{\rho} (\tau_b - 2\bar{\tau})$
Sediment transport (15)	s_0	$-\frac{\partial has_1}{\partial t} - \alpha\omega(s_0 - s_*)$
Moment for sediment (16)	s_1	$hu_1 \frac{\partial(s_0 + as_1)}{\partial x} - 3\alpha\omega(s_0 - s_*)$ $-\frac{12}{h} \varepsilon_s s_1 + 6\omega(s_0 + as_1)$

control volumes, and, of course, over the whole calculation domain. These characteristics exist for any number of grid points. Thus, even the coarse-grid solution exhibits exact integral balances. To obtain stable solutions, the stagger grid (Fig. 3) is used. The scalar variables, such as water depth and concentrations, are on the main grid point i [Fig. 3(b)]. Vector variables, such as velocities, are on the middle point $i \pm 1/2$ between main points i and $i - 1$ [Fig. 3(c)].

Discretization Equation

The integration of (24) over the control volume shown in Fig. 3 [for u_0 and u_1 Fig. 3(c) is used; for s_0 and s_1 Fig. 3(b) is used] would give

$$\frac{(h\phi)_i - (h\phi)_i^0}{\Delta t} \Delta x(i) + (hu_0\phi)_{i+1/2} - (hu_0\phi)_{i-1/2} = (S_c + S_p\phi_i)\Delta x(i) \quad (25)$$

where S_ϕ is linearized as $S_\phi = S_c + S_p\phi_i$, S_c and S_p are coefficients of linearized source term treatment. The superscript 0 denotes the old values, i.e., the values in last time step. $\Delta x(i)$ is the length of the control volume. For h , s_0 , and s_1 , $\Delta x(i) = \Delta x_i^h$; for u_0 and u_1 , $\Delta x(i) = \Delta x_i^u$.

In a similar way, we can integrate the continuity equation [(1)] over the control volume [Fig. 3(b)], and obtain

$$\frac{h_i - h_i^0}{\Delta t} \Delta x(i) + (hu_0)_{i+1/2} - (hu_0)_{i-1/2} = 0 \quad (26)$$

Eqs. (25) and (26) $\cdot \phi_i$ result in

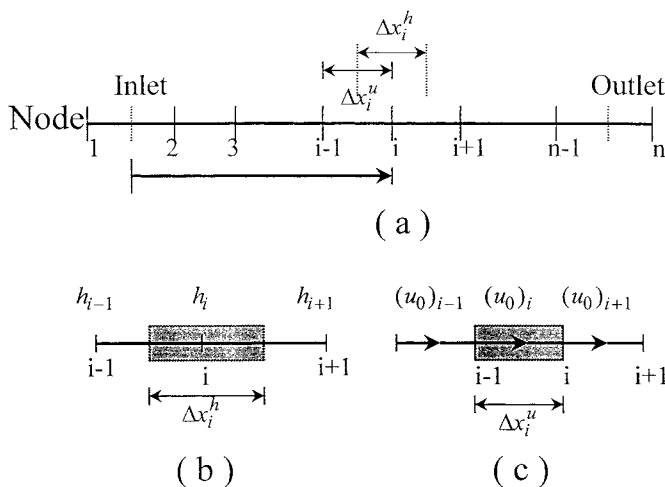


FIG. 3. Discretization Grid Sketch

$$\left[\frac{(h\phi)_i - (h\phi)_i^0}{\Delta t} - \frac{h_i - h_i^0}{\Delta t} \phi_i \right] \Delta x(i) + [(hu_0)_{i+1/2}\phi_{i+1/2} - (hu_0)_{i-1/2}\phi_{i-1/2}] - [(hu_0)_{i-1/2}\phi_{i-1/2} - (hu_0)_{i-1/2}\phi_i] = (S_c + S_p\phi_i)\Delta x(i) \quad (27)$$

After simplification (Patankar 1980), (27) can be written as

$$-a_{i-1}\phi_{i-1} + a_i\phi_i - a_{i+1}\phi_{i+1} = S_u \quad (28)$$

where

$$a_{i+1} = \max[-(hu_0)_{i+1/2}, 0]; \quad a_{i-1} = \max[(hu_0)_{i-1/2}, 0] \quad (29a, b)$$

$$a_i = a_{i+1} + a_{i-1} + \frac{h_i^0}{\Delta t} \Delta x(i) - S_p\Delta x(i) \quad (29c)$$

$$S_u = S_c\Delta x(i) + \frac{(h\phi)_i^0}{\Delta t} \Delta x(i) \quad (29d)$$

All variables, except water depth, can be solved by (28). Water depth can be obtained directly from continuity equation. Assuming the profile for h between two nodes is linear, (26) becomes

$$\frac{h_i - h_i^0}{\Delta t} \Delta x_i^h + \frac{1}{2} (h_{i+1} + h_i)(u_0)_{i+1} - \frac{1}{2} (h_i + h_{i-1})(u_0)_i = 0 \quad (30)$$

Eq. (30) is further written as

$$-a_{i-1}h_{i-1} + a_i h_i - a_{i+1}h_{i+1} = S_u \quad (31)$$

where

$$a_{i-1} = 0.5(u_0)_i; \quad a_{i+1} = -0.5(u_0)_{i+1}$$

$$a_i = \frac{\Delta x_i^h}{\Delta t} + \frac{(u_0)_{i+1} - (u_0)_i}{2}; \quad S_u = \frac{\Delta x_i^h}{\Delta t} h_i^0$$

The discretization equations [(28) and (31)] have the form of the well-known tridiagonal matrix. They can be solved by the Thomas algorithm.

Computational Procedure

To start the computations, the initial conditions (i.e., the values of h , u_0 , u_1 , s_0 , and s_1 at $t = 0$) are given for all grid points. The boundary conditions (i.e., the values of u_0 , u_1 , s_0 , and s_1 at inlet and the value of h at outlet) are given for all time steps. The values of all variables at interior nodes at the end of the time interval Δt are computed as follows:

1. The values of $(u_0)_i$ and $(u_1)_i$ at the interior nodes ($i = 2, \dots, N - 1$) are computed by using (28). The values at the boundaries ($i = 1$ and $i = N$) are computed by using the boundary conditions.
2. The values of h_i at the interior nodes ($i = 2, \dots, N - 1$) are calculated by using (31). At the boundaries, values are calculated by using the given conditions.
3. Steps 1 and 2 are repeated until the convergent solutions for water flow are obtained.
4. The values of $(s_0)_i$ and $(s_1)_i$ at the interior nodes are also obtained from (28). Their values at the boundaries are obtained from boundary conditions.
5. After obtaining the results of sediment concentrations, the bed variations are calculated using (18). Thereby, the bed elevation is modified.
6. To calculate hydraulic factors in the modified channel for the next time interval, Steps 1–3 are repeated. Then Steps 4 and 5 are repeated to obtain the new bed elevation. The computation will be determined when the required time is reached.

From this computational procedure, the calculations of hydraulic and sediment factors are not coupled. In most situations, this uncoupled calculation does not affect the accuracy of bed variations because in a short time interval the erosion or deposition of the channel has a negligible influence on hydraulic factors.

SENSITIVITY ANALYSIS OF COEFFICIENTS

Because of the complexity of sediment transport, it is difficult, even impossible, to eliminate coefficients in a mathematical model. Thus, if a model has as few coefficients as possible, and they can be easily determined through theoretical foundation and field data, the model is considered to be significant and useful.

The coefficients to be calibrated in this model are α and k . Both coefficients are related to sediment transport. To determine the influence of α and k on sediment transport, a number of numerical experiments have been made in a rectangular channel (length = 24.8 m, width = 0.5 m) with an erodible bed. The channel has a deep reach with a length of 4.8 m, a depth of 0.15 m, and a side slope of 1:8. The channel bed slope is 0.0001. The bed material is 0.16 mm. The discharge and suspended sediment concentration at the inlet are 0.09945 m³/s and 0.1612 kg/m³, respectively. In each numerical experiment one coefficient (α or k) is fixed while the other varied.

Influence of Coefficient α

To understand the influence of coefficient α on the results, three different experiments were run with $k = 0.007$ and $\alpha = 5.0, 10.0$, and 15.0 . Fig. 4 shows the results after 10 h of the simulation. It can be seen that both deposition and erosion are faster with increasing values of α . These phenomena are consistent with the physical meaning of the coefficient. Because α primarily expresses the ratio of the concentration near the bed and the average concentration of the cross section, it can be easily understood that both deposition and erosion are favored by greater α .

From the derivation of sediment transport equation, the physical meaning of coefficient α is the ratio of sediment concentration near the bed to the average concentration of the cross section. Thus, coefficient α should be greater than unity. There are many factors affecting coefficient α . The most important parameter may be sediment size. The sediment concentration is more uniformly distributed for smaller sediment size. This results in a smaller value for coefficient α . Contrarily, the bigger the sediment size, the more nonuniform distribution will be. A bigger value for coefficient α is expected in this situation. In application, coefficient α can be approximately equal to the ratio of s_b/s_0 . When the distributions of velocity and suspended sediment concentration along the depth are given, average concentration s_0 can be computed by using (9) and concentration near the bed s_b can be read from the concentration curve. When there is lack of the velocity and concentration distributions, coefficient α is roughly estimated by the sediment size, and then calibrated by field data. In either case, calibration is needed in the model application.

Influence of Coefficient k

To find the influence of k on the results, three numerical experiments were made with $\alpha = 14.0$ and $k = 0.006, 0.008$, and 0.01 , respectively. Fig. 5 shows the results after 10 h of the numerical test. As expected, in the upstream end, deposition occurred for $k = 0.006$ and erosion occurred for $k = 0.01$, whereas equilibrium was observed for $k = 0.008$. In the deeper reach, more deposition was observed for larger k values as more sediment was carried into the deeper reach. In the down-

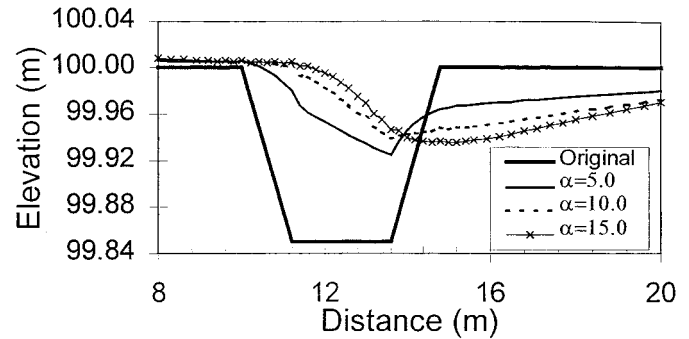


FIG. 4. Bed Elevations for Different α (after 10 h)

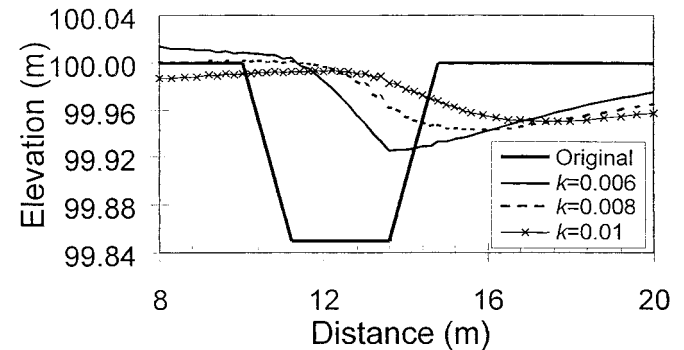


FIG. 5. Bed Elevations for Different k (after 10 h)

stream end, the larger the value of coefficient k , the more the erosion. These phenomena are consistent with the nature of coefficient k .

Coefficient k is a reflection of the capacity of sediment transport by flow. It is related to both hydraulic and sediment factors. It is inversely proportional to the size of sediment and water depth, and directly proportional to flow velocity. In application, coefficient k may be estimated by (23), if the equilibrium suspended sediment concentration is known, i.e., $s_* = s_0$. When modeling sediment transport in a river, it is possible to find a time period when the river is under equilibrium condition. The estimated k value is considered to be appropriate to simulate sediment transport in the river as there is no extreme changes in the inputs of flow and sediment. Although k is estimated by (23), calibration is still required when one uses the model.

COMPARISON WITH EXPERIMENTAL DATA

The numerical results of the model are compared with the experimental data produced by Guy et al. (1966), Rijn (1980), and Lee and Yu (1991). The comparison of this model with the HEC-6 model (cited from Tingsanchali and Supharatid 1996) is also presented.

Comparison with Lee and Yu's Data

The numerical results are first assessed by comparing them with data from Lee and Yu (1991). The experiment (Run NS2) was carried out in a flume with a length = 20 m, width = 0.2 m, and depth = 0.6 m. The average grade of suspended sediment was 0.05 mm. The water depths were measured at distances of 8.3, 9.3, 12.3, and 13.3 m, and the average suspended sediment concentrations were measured at distances of 8.3, 9.3, 10.3, 11.3, 12.3, and 13.3 m. The comparison of computed and measured water depth is shown in Fig. 6. As can be seen from this chart, the numerical results agree very well with the experimental results.

To calculate sediment concentrations, the relative coeffi-

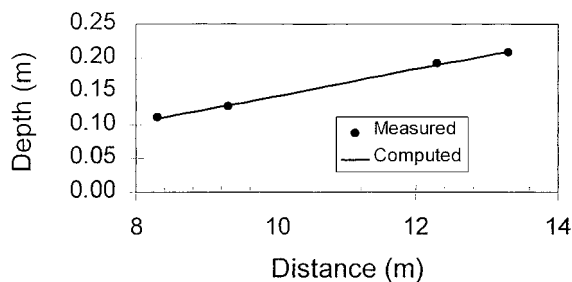


FIG. 6. Comparison of Computed and Measured Flow Depths Using Lee and Yu's (1991) Data

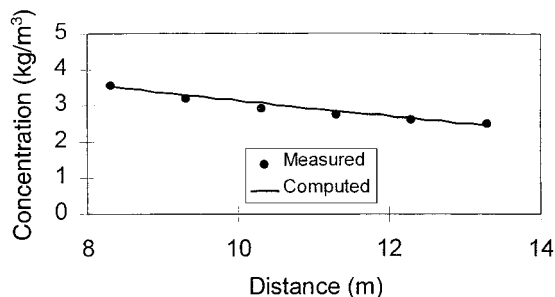


FIG. 7. Comparison of Computed and Measured Concentrations Using Lee and Yu's (1991) Data

coefficients (α and k) in the sediment equations are assigned the values of 1.5 and 0.02, respectively. Calculated and measured sediment concentration profiles along the flume are shown in Fig. 7. The agreement is reasonably good through most of the flow section; there is a slight deviation in the middle section.

Comparison with Guy et al.'s Data

A large quantity of experimental data from Guy et al. (1966) are used to further verify the suspended load computation of the model. The initial objective of the experiments was to obtain mean values of hydraulic and sediment variables for the large range of conditions commonly found in natural streams and artificial channels. The mean values were measured only when equilibrium conditions were established. Equilibrium within the flume was defined as a condition of statistically uniform velocity, concentration, and slope with respect to both time and space.

A total of 339 experiments were made in 0.61- and 2.44-m-wide recirculating flumes. Most of the measurements were collected in a flume 2.44 wide, 0.61 m deep, and 45.5 m long. The flow could be varied from 0 to 0.62 m³/s and the slope could be adjusted from 0 to 1.5%. The average diameters of sediment were from 0.19 to 0.93 mm. Other measurements were conducted in a flume 0.61 m in width, 0.76 m in depth, and 18.3 m in length. The flow rate was adjusted from 0 to 0.23 m³/s, and the slope, from horizontal to 10%. The average diameters of sediment were from 0.32 to 0.33 mm.

Among the 339 runs, only 143 runs contain enough information to verify the model. Deducting the experiments in steep flow conditions, 115 runs are used in the comparison. Among these 115 runs, 77 runs were made in the 2.44-m-wide flume and 38 runs were made in the 0.61-m-wide channel.

To calculate the equilibrium sediment concentration, coefficient α is given the value of 12 for both flumes. Coefficient k is given the values of 0.015 for the 2.44-m-wide flume and 0.016 for the 0.61-m-wide flume, respectively. Different k values are used because the sediment composition in a 0.61-m-wide flume is smaller than that in a 2.44-m-wide flume. The values of k and α are kept constant for all experiments in these two flumes. The comparisons of computed and measured sed-

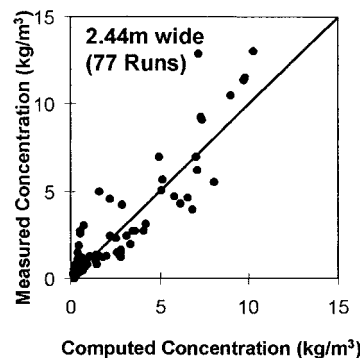


FIG. 8. Relationship between Computed and Measured Sediment Concentrations (2.44 m wide) Using 77 Runs from Guy et al.'s (1966) Data

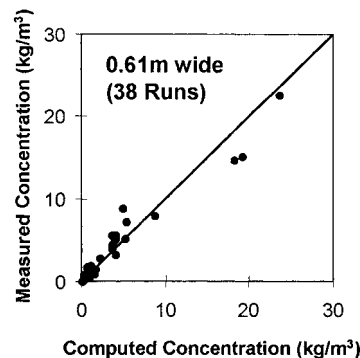


FIG. 9. Relationship between Computed and Measured Sediment Concentrations (0.61 m wide) Using 38 Runs from Guy et al.'s (1966) Data

iment concentrations at equilibrium are shown in Figs. 8 and 9. The horizontal coordinate is the computed concentrations and the vertical coordinate is the measured concentrations. The perfect fit between computed and measured concentrations is on the 45° lines. It can be seen from these figures that there is a very good agreement. Also, it is found that the results for the 0.61-m-wide flume is better than that for the 2.44-m-wide channel. This may be due to the more uniform sediment size used in the 0.61-m-wide flume test.

Comparison with Rijn's Data

The present model is applied to a flume experiment to simulate the bed deformation of a trench. The experiment was carried out by Delft Hydraulics Laboratory (Galappatti and Vreugdenhil 1985; Rijn 1986), in a uniform flow with a fully developed sediment concentration profile over a gentle-sided (1:10) trench in a 30-m-long, 0.5-m-wide, and 0.7-m-deep flume. The mean flow velocity and the flow depth were 0.51 m/s and 0.39 m, respectively. The bed consisted of fine sand ($d_{50} = 0.16$ mm). The settling velocity of the suspended sediment was 0.013 m/s. During the experiment, the equilibrium conditions were maintained in the upstream end. Under this situation, the equilibrium unit-wide suspended sediment rate was 0.03 kg/m/s and the equilibrium suspended sediment concentration of the cross section in the upstream end was 0.1508 kg/m³. The trench was 0.15 m deep initially and deformation was measured.

The coefficients α and k in the present model can be approximately estimated based on the conditions of the experiment and the distributions of measured velocity and suspended sediment concentration along the depth. Because the equilibrium conditions were maintained at the upstream end through the experiment, this means that the sediment-carrying capacity was equal to the suspended sediment concentration of the cross

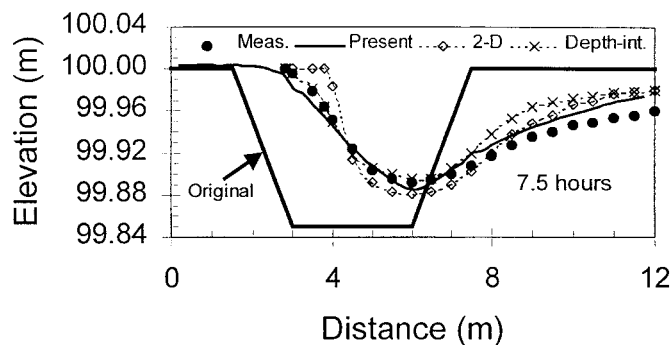


FIG. 10. Computed and Measured Bed Elevations after 7.5 h (Galappatti and Vreugdenhil 1985; Rijn 1986)

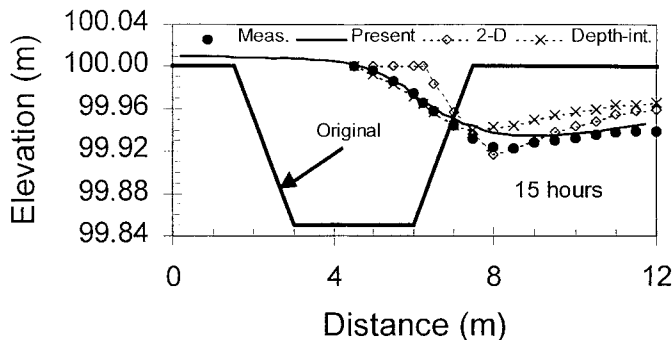


FIG. 11. Computed and Measured Bed Elevations after 15 h (Galappatti and Vreugdenhil 1985; Rijn 1986)

section. From the sediment-carrying capacity formula [(23)], a value of 0.0075 for coefficient k is obtained. The coefficient α can be estimated by the ratio of sediment concentration near the bed s_b to the average concentration over the cross section. From (9) and the concentration and velocity profiles provided (Rijn 1986), a value of 14.5 is estimated for coefficient α . In the simulation, the values of 14.0 and 0.007 for coefficients α and k , respectively, are used.

In the numerical simulation, the grid length for different reaches of the flume is varied. In the upstream and downstream reaches, $\Delta x = 0.5$ m. Finer grids ($\Delta x = 0.25$ m) are used in the deep reach. In the transition regions, from the upstream end to the deep reach and from the deep reach to the downstream end, the value from 0.3 to 0.4 for Δx is used. The bed elevations are modified every 15 min. The comparisons of the computed bed elevation profiles by this model after 7.5 and 15 h with the experimental data and the computed results from two other models (i.e., depth-integrated model and full 2D numerical model), in the paper by Galappatti and Vreugdenhil (1985), are presented in Figs. 10 and 11. The computed results from the present model agree well with the experimental data.

Comparison with HEC-6 Model

The HEC-6 model is developed by the Hydrologic Engineering Center (HEC) of the U.S. Army Corps of Engineers to simulate a long-term average of scour and deposition in rivers and reservoirs. Tingsanchali and Supharatid (1996) used their experimental data to make an investigation and analysis for the HEC-6 model. The experiment (Run No. N1) was carried out under subcritical flow condition in a rectangular flume with a test section 12 m long and 0.6 m wide. A nearly uniform sediment of $d_{50} = 0.0009$ m was used as bed material. A constant inflow of $0.054 \text{ m}^3/\text{s}$ without sediment supply was fed at the upstream end of the flume. The initial bed has a uniform thickness of 20 cm for a distance of 6 m and a downward step of 10 cm to a thickness of 10 cm in the remaining downstream

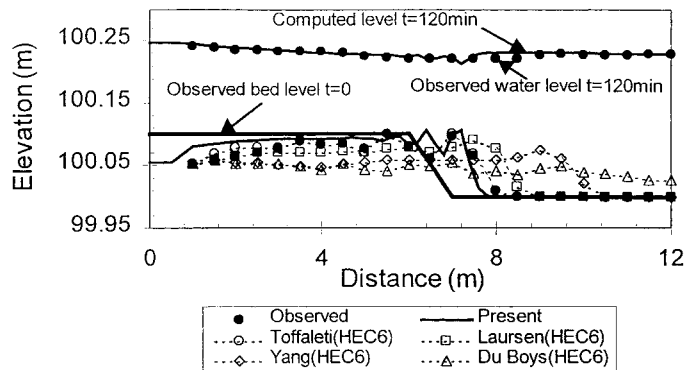


FIG. 12. Comparison of the Present Model with Experimental Data and Numerical Models by Laursen, Toffaleti, Yang, and Du Boys (All Used HEC-6 Model) (Tingsanchali and Supharatid 1996)

portion of 6 m. Toffaleti, Laursen, and Yang and Du Boys' formulas [see Tingsanchali and Supharatid (1996)], were used as the sediment transport equation in the HEC-6 model to simulate the bed variations in their paper. The computed bed elevations by the HEC-6 model, present model, and the observed results are compared and plotted in Fig. 12. The computed bed profiles based on the HEC-6 model (the Toffaleti and Laursen formulas were used) and the present model agree well with the experimental data, whereas those computed using the Yang and Du Boys formulas as the sediment transport equation of HEC-6 model considerably lead the experimental results.

CONCLUSIONS

1. A 1D numerical model based on the depth-averaged and moment equations has been developed to calculate the variation of bed elevation in alluvial channels. Hence, the suspended sediment concentrations can be directly solved from the set of equations. The use of the concept of sediment-carrying capacity and the treatment of the vertical sediment diffusion term avoid the difficulty of calculating the reference concentration near the bed and the problem of selecting a sediment transport function. These are two problems that most existing models cannot escape.
2. Both coefficients α and k have physical and theoretical significance. They can be approximately estimated through measured data such as sediment size, equilibrium concentration, distributions of velocity, and suspended sediment concentration. However, calibration and verification are required.
3. The comparisons of the computed and experimental data show that the proposed model yields reliable results in calculating the sediment concentration. The simulated bed elevation profiles show a very good agreement with the measured results. It is expected that the proposed model can be applied to various hydraulic projects due to its simplicity and good accuracy.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

- a = coefficient, $a = u_1/(3u_0)$;
- a_{i-1}, a_i, a_{i+1} = coefficients of discretization equations;
- C = Chézy coefficient;
- d_{50} = average sediment diameter (m);
- g = gravitational acceleration (m/s^2);
- H = water level (m);
- h = water depth (m);
- k = sediment-carrying capacity coefficient;
- m = sediment-carrying capacity constant;
- n = Manning coefficient;
- S_u = right-hand term of discretization equations;
- S_ϕ = source term, $S_\phi = S_c + S_p \phi_i$;
- s = sediment concentration (kg/m^3);
- s_b = sediment concentration near bed (kg/m^3);
- s_{b*} = sediment-carrying capacity near bed (kg/m^3);
- s_0 = average suspended sediment concentration (kg/m^3);
- s_1 = half of concentration difference near bed and at surface (kg/m^3);
- s_* = sediment-carrying capacity (kg/m^3);
- t = time (s);
- u = flow velocity in longitudinal direction (m/s);
- u_0 = depth-averaged velocity (m/s);
- u_1 = longitudinal velocity in excess of depth-averaged velocity at water surface (m/s);
- u_* = shear-stress velocity (m/s);
- \bar{v}_t = depth-averaged eddy viscosity (m^2/s);
- w = vertical flow velocity (m/s);
- w_H, w_b = vertical velocities at surface and bed, respectively (m/s);
- x = horizontal distance along channel (m);
- z = vertical distance from bed (m);
- z_b = bed elevation (m);
- α = sediment coefficient;
- α_1, α_2 = coefficients;
- γ' = dry specific weight of bed materials (t/m^3);
- Δt = time interval (s);
- $\Delta x(i)$ = space interval (m);
- Δx_i^h = length of control volume for h, s_0 , and s_1 (m);
- Δx_i^u = length of control volume for u_0 and u_1 (m);
- ε_z = diffusion coefficient (m^2/s);
- $\eta = (z - z_b)/h$;
- κ = constant of Von Kármán;
- λ = mixing length coefficient;
- ρ = water density (t/m^3);
- $\bar{\tau}$ = depth-averaged shear stress (F/m^2);
- τ_b = bed shear stress (F/m^2);
- ϕ = general variable, such as velocities and concentrations;
- ω = sediment settling velocity (m/s); and
- ω_b = sediment settling velocity near bed (m/s).

Subscript

- i = node along x -axis.