

# BENCHMARK PROBLEM -1- TSUNAMI RUNUP ONTO A PLANE BEACH

Zygmunt Kowalik, Juan Horrillo and Edward Kornkven

*Institute of Marine Science, University of Alaska Fairbanks  
Fairbanks, AK 99775, USA*

*E-mail: ffzk@ims.uaf.edu*

**Abstract** The problem description and setup of Benchmark Problem 1 (BM1) can be found in Part I Section 2 of this proceedings. A detailed analytical solution of the problem is described in Carrier, Wu and Yeh.<sup>1</sup>

To solve BM1 three approaches have been carried out:

- 1) First order approximation in time
- 2) Second order approximation in time (leap-frog)
- 3) Full Navier-Stokes (FNS) approximation aided by the Volume of Fluid (VOF) method to track the free surface.

Approach 1) and 2) use one-dimensional nonlinear shallow water (NLSW) wave theory. The finite difference solution of equation of motion and the continuity are solved on a staggered grid. Both methods have second order approximation in space. The FNS-VOF approach has been used to visualize differences against the NLSW approaches and analytical solution. The FNS equation includes the vertical component of velocity/acceleration and some differences are expected. This method solves two-dimensional transient incompressible fluid flow with free surface. The finite difference solution of the incompressible FNS equations are obtained on a rectilinear mesh.

## 1. Brief Description of the Methods and their Numerical Schemes

### 1.1 First Order Method

Equations of motion and continuity read

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial \zeta}{\partial x} + \frac{1}{\rho D} r u |u| = 0 \quad \text{and} \quad \frac{\partial \zeta}{\partial t} + \frac{\partial u D}{\partial x} = 0, \quad (1)$$

where  $\rho$  is the water density,  $u$  is the vertically averaged particle velocity,  $\zeta$  is the sea level,  $D = (\zeta + H)$  is the total depth,  $H$  is the mean water depth,  $r$  is the friction coefficient and  $g$  is the gravity acceleration. The numerical solution is usually searched by using the one-time-level numerical scheme, Kowalik and Murty.<sup>7</sup> The numerical scheme is constructed as follow

(Kowalik and Murty<sup>8</sup>):

$$\begin{aligned} u_j^{m+1} &= u_j^m - \frac{gT}{h}(\zeta_j^m - \zeta_{j-1}^m) - \frac{u_p^m T}{h}(u_j^m - u_{j-1}^m) \\ &\quad - \frac{u_n^m T}{h}(u_{j+1}^m - u_j^m) - \frac{2T}{\rho(D_{j-1}^m + D_j^m)} r u_j^m |u_j^m| \\ \zeta_j^{m+1} &= \zeta_j^m - \frac{T}{h}(flux_{j+1}^{m+1} - flux_j^{m+1}), \end{aligned} \quad (2)$$

where  $flux_j = u_p^{m+1} \zeta_{j-1}^m + u_n^{m+1} \zeta_j^m + u_j^{m+1} \frac{(H_j + H_{j-1})}{2}$ ,  $u_p = 0.5(u_j + |u_j|)$  and  $u_n = 0.5(u_j - |u_j|)$ .  $T$  is the time step,  $h$  is the space step. Indices  $m$  and  $j = 1, 2, 3, \dots, n-1$  stand for the time stepping and horizontal coordinate points, respectively. For the runup condition the following steps are taken when the dry point ( $j_{wet} - 1$ ) is located to the left of the wet point  $j_{wet}$ , thus: if  $(\zeta^m(j_{wet}) > -H(j_{wet} - 1))$ , then  $u_{j_{wet}}^m = u_{j_{wet}+1}^m$ , see Kowalik and Murthy.<sup>8</sup>

## 1.2 Leap Frog Method

Equation of motion and continuity are expressed in flux form as

$$\frac{\partial M}{\partial t} + \frac{\partial M^2}{\partial x} + gD \frac{\partial \zeta}{\partial x} + \frac{gn^2}{D^{7/3}} M |M| = 0 \quad \text{and} \quad \frac{\partial \zeta}{\partial t} + \frac{\partial M}{\partial x} = 0, \quad (3)$$

where  $M = uD$  is the water transport and  $n$  is the Manning's roughness coefficient. The numerical scheme is constructed on a space-time staggered grid having second order of approximation in space and time, see Imamura *et al.*<sup>4</sup>

The two-time-level numerical scheme reads

$$\begin{aligned} \zeta_j^m &= \zeta_j^{m-1} - \frac{T}{h}(M_j^{m-1/2} - M_{j-1}^{m-1/2}) \\ M_j^{m+1/2} &= \frac{1}{(1 + \mu_x)} [(1 - \mu_x) M_j^{m-1/2} - \frac{gD_r T}{h} (\zeta_{j+1}^m - \zeta_j^m) \\ &\quad - \frac{T}{h} (\lambda_1 \frac{(M_{j+1}^{m-1/2})^2}{D_{M_{j+1}}^{m-1/2}} + \lambda_2 \frac{(M_j^{m-1/2})^2}{D_{M_j}^{m-1/2}} + \lambda_3 \frac{(M_{j-1}^{m-1/2})^2}{D_{M_{j-1}}^{m-1/2}})] \end{aligned} \quad (4)$$

where  $\mu_x$  is a friction term factor,  $D_M$  is the total depth at  $M$  points, and  $D_r$  is the total depth which depends of the sea level and depth of the neighboring cells. Parameters  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  are the up-down wind's switches used in the nonlinear term.  $\mu_x$  and  $D_M$  are defined as  $\mu_x = \frac{gn^2 T}{2D_r^{7/3}} |M_j^{m-1/2}|$  and  $D_{M_j}^{m-1/2} = \frac{1}{4}(D_j^{m-1} + D_{j+1}^{m-1} + D_j^m + D_{j+1}^m)$ , respectively.

### 1.3 Full Navier-Stokes Approximation and VOF Method

Equation of continuity for incompressible fluid and the momentum equation

$$\nabla \cdot \vec{u} = 0 \quad \text{and} \quad \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{u} + \vec{g} \quad (5)$$

are solved in the rectangular system of coordinates. Where  $\vec{u}(x, y, t)$  is the instantaneous velocity vector,  $\rho$  is the fluid density,  $p$  is the scalar pressure,  $\nu$  is the kinematic viscosity,  $\vec{g}$  is the acceleration due to gravity and  $t$  is the time. Solution of the equations is searched using the two-step method (Chorin<sup>2</sup> and Harlow & Welch.<sup>3</sup>). The time discretization of the momentum equation is given by

$$\frac{\vec{u}^{m+1} - \vec{u}^m}{T} = -(\vec{u} \cdot \nabla) \vec{u}^m - \frac{1}{\rho^m} \nabla p^{m+1} + \nu \nabla^2 \vec{u}^m + \vec{g}$$

and it is broken up into two steps as follow:

$$\frac{\vec{u} - \vec{u}^m}{T} = -(\vec{u} \cdot \nabla) \vec{u}^m + \nu \nabla^2 \vec{u}^m + \vec{g} \quad (6), \quad \frac{\vec{u}^{m+1} - \vec{u}}{T} = -\frac{1}{\rho^m} \nabla p^{m+1} \quad (7)$$

Eq. (7) and the continuity equation,  $\nabla \cdot \vec{u}^{m+1} = 0$ , can be combined into a single equation (Poisson equation) for the solution of the pressure as

$$\nabla \cdot \left[ \frac{1}{\rho^m} \nabla p^{m+1} \right] = \frac{\nabla \cdot \vec{u}}{T}.$$

The fluid free surface is described by the discrete VOF method, see Nichols and Hirt.<sup>5,6</sup>

## 2. Discussion and Conclusions

Figs. 1, 2 and 3 summarize results of BM1. The simple velocity extrapolation used in the first order method seems to follow the shoreline evolution as prescribed by the NLSW analytical solution. Sea level and velocity profile match very well with the analytical solution. The extrapolation of the velocity from the immediate wet cell to the dry cell facilitates runup improving the timing. The leap frog method does well in predicting the analytical solution of the shoreline evolution, sea level and velocity profile as well. However, due to the small difference in timing, some discrepancy in the velocity profile can be seen, i.e. at time 220s. The FNS-VOF method gives a frame of reference to validate the NLSW solutions. Some differences in wave profile, shore line evolution and timing are quite plausible, since FNS approximation allows vertical fluid velocity/acceleration while the NLSW theory does not. From Figs. 1 and 3, it is clear that dispersion effects are important. NLSW and analytical solutions underestimate the runup and

overestimate the rundown. Timing of maximum runup and rundown occur slightly earlier in the NLSW solutions.

### References

1. G. Carrier Tai Tei Wu, and H. Yeh, "Tsunami runup and draw-down on a plane beach", *Journal of Fluid Mechanics*. **475**:79-99, 2003
2. A.J. Chorin. Numerical solution of the Navier-Stokes equations. *Math. Comp.*, **22**:745-762, 1968.
3. F.H. Harlow and J.E. Welch. "Numerical calculation of time-dependent viscous incompressible flow of fluid with a free surface". *The Physics of Fluids*, **8**:2182-2189, 1965.
4. F.Imamura, C. Goto, Y. Ogawa and N. Shuto. "Numerical Method of Tsunami Simulation with the Leap-Frog Scheme", *IUGG/IOC Time Project Manuals*, 1995.
5. B. D. Nichols and C. W. Hirt, "Method for Calculating Multi-Dimensional, Transient Free Surface Flow Past Bodies," *Proc. of the 1st Int. Conf. Num. Ship Hydrodynamics*, Gaithersburg, Maryland, 1975.
6. B. D. Nichols, C. W. Hirt and R. S. Hotchkiss, "SOLA-VOF: A solution Algorithm for Transient Fluid Flow with multiple Free Boundaries", *LA-8355, Los Alamos National Laboratory*, 1980.
7. Z. Kowalik, and T. S. Murty, *Numerical Modeling of Ocean Dynamics*, World Scientific, :481, 1993.
8. Z. Kowalik and T. S. Murty, "Numerical Simulation of Two-Dimensional Tsunami Runup", *Marine Geodesy*, **16**:87-100, 1993.

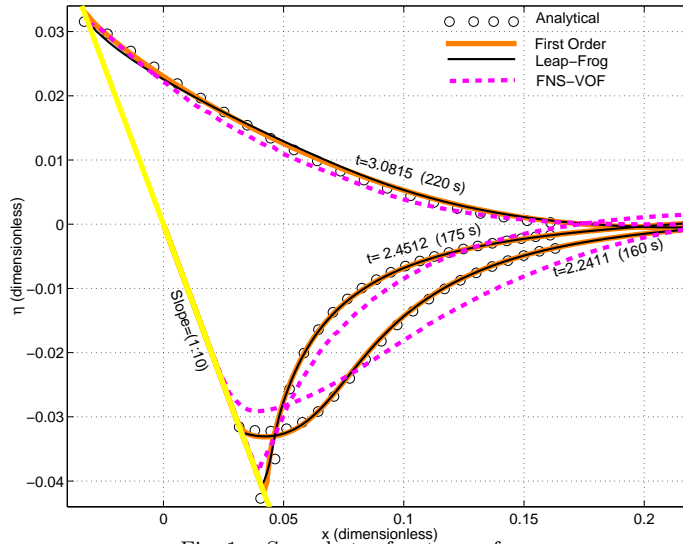


Fig. 1. Snapshots of water-surface

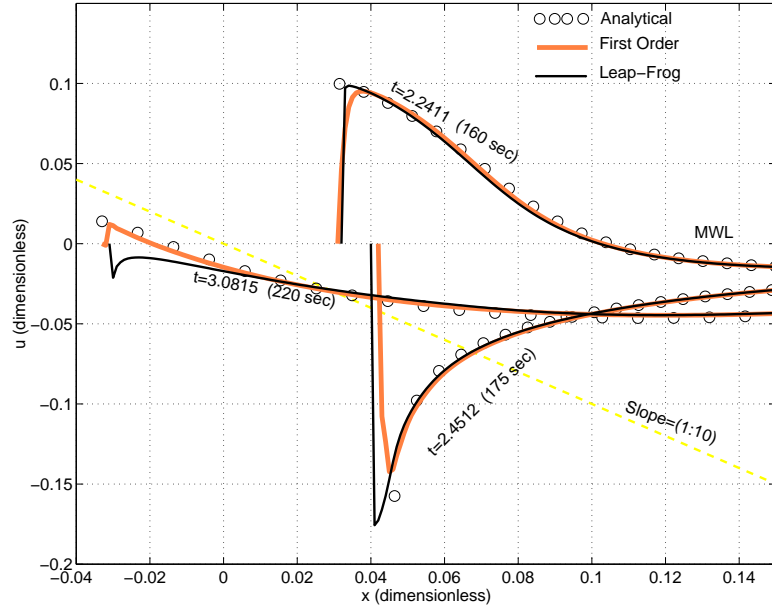


Fig. 2. Snapshots of velocity profiles

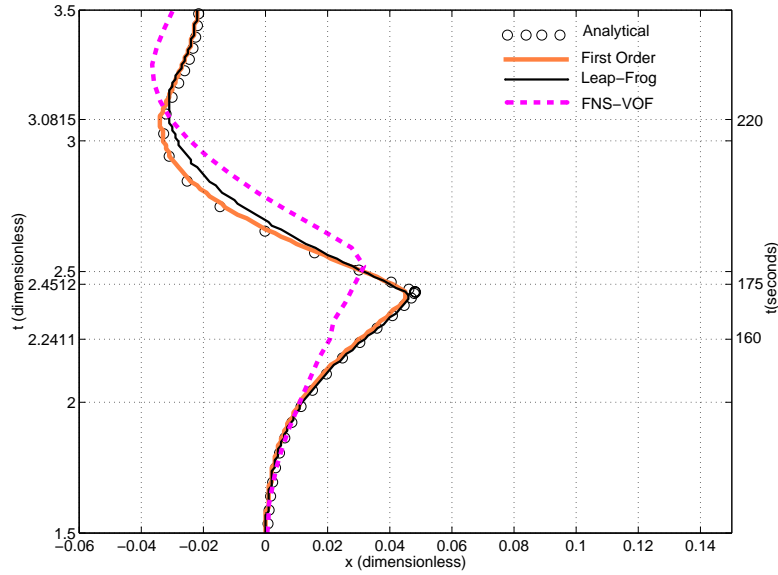


Fig. 3. Temporal and spatial variation of the shoreline