Coastal Modelling System Non-Equilibrium Sediment Transport Model (NET)

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Outline

- Introduction
  - CMS and NET Overview
  - Equilibrium vs. Non-equilibrium sediment transport
  - Advantages of Non-equilibrium
- Model Equations: An overview
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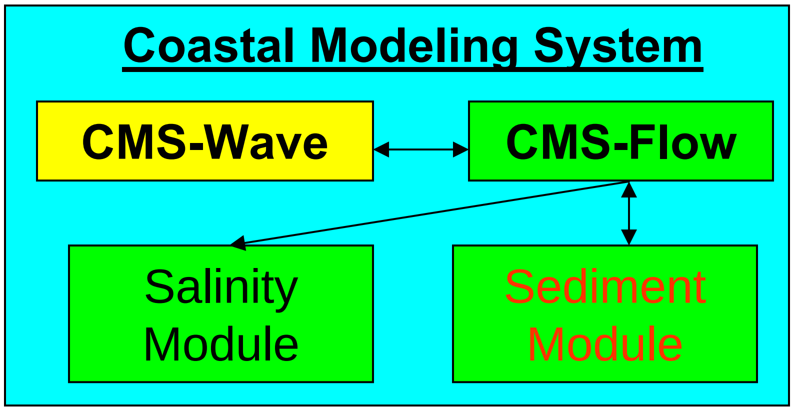
  - Adaptation length
  - Bed-slope coefficient
  - Sediment diffusivity
  - Total load correction factor
- Avalanching
- Numerical implementation
- Questions

Introduction: CMS Overview

- Overview



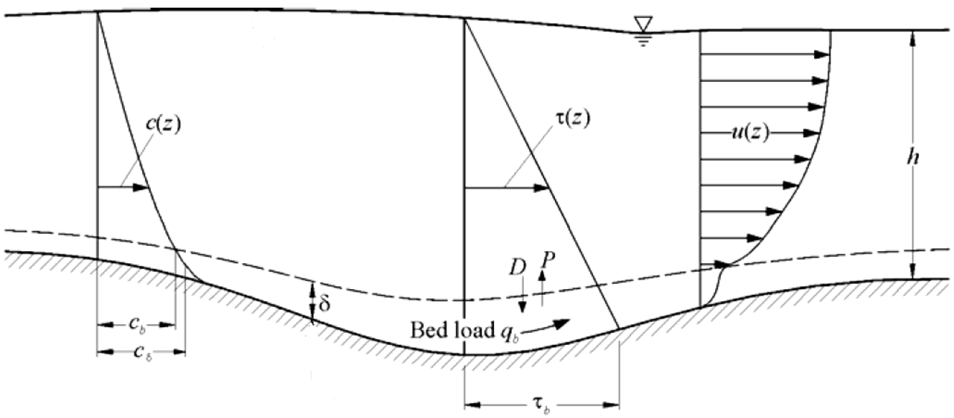
- Coupling
  - Wave-Flow
    - Radiation stress
    - Wave height, period and direction
  - Flow-Wave
    - Water elevation
    - Currents
    - Updated bathymetry

- Sediment Module

1. Total load
   - Sediment balance Eq.
2. Equilibrium Transport (ET)
   - Advection-Diffusion Eq.
     - Suspended load
   - Bed change Eq.
     - Morphology change
     - Bed-slope effect
     - Bed load transport
3. Non-Equil. Transport (NET)
   - Advection-Diffusion Eq.
     - Total load
   - Bed change Eq.
     - Morphology change
     - Bed-slope effect

Introduction: NET Overview

- 2D depth-averaged
- Features (processes)
  – Advection
  – Diffusion
  – Erosion and deposition
  – Bed-slope effects
  – Avalanching
- Definition of variables



Introduction: Equilibrium vs. Non-Equilibrium

- Equilibrium sediment transport
  - Assume local instantaneous equilibrium for bed-load transport or total-load
  - Bed change is determined by mass balance equation (Exner equation)
- Non-equilibrium sediment transport models
  - Do not assume any sediment transport load to be in equilibrium
  - Bed change is proportional to difference between local and equilibrium transport rates

Advantages of NET

- Considers temporal and spatial lags between flow and sediment transport
- Can easily handle constrained sediment loading (over- or under-loading)
- Hard-bottom problem is no problem
- Can model suspended and bed load separately or combined as bed-material or total load
- More stable than equilibrium sediment transport

Model Equations: Quick Overview

- Advection-diffusion

$$\frac{\partial}{\partial t}\left(\frac{dC_t}{\beta_t}\right)+\frac{\partial(dUC_t)}{\partial x}+\frac{\partial(dVC_t)}{\partial y}=\frac{\partial}{\partial x}\left[Kd\,\frac{\partial(r_s C_t)}{\partial x}+\right]+\frac{\partial}{\partial y}\left[Kd\,\frac{\partial(r_s C_t)}{\partial y}\right]+\alpha_t w_f\,(C_{t*}-C_t)$$

C_{t*} = Concentration Capacity

α_t = Total Load Adapt. Coeff.

w_f = Sediment Fall Velocity

r_s = Fraction of suspended load

β_t = Total Load Correction Factor

K = Diffusion Coefficient

S_b = Bed-slope term

D_s = Bed-slope coefficient

- Bed change equation

$$(1-p_m^{'})\frac{\partial h}{\partial t}=\alpha_t\,\omega_f\,(C_{t*}-C_t)+S_b$$

- Bed-slope term

$$S_b=\frac{\partial G_x}{\partial x}+\frac{\partial G_y}{\partial y}=\frac{\partial}{\partial x}\left[D_s(1-r_s)d\left|U\right|C_t\,\frac{\partial h}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_s(1-r_s)d\left|U\right|C_t\,\frac{\partial h}{\partial y}\right]$$

Concentration Capacity

- The concentration that would be achieved under steady-state and equilibrium conditions
- Formula

$$C_{t*} = \frac{Q_t}{U_c d}$$

$$Q_t = f_s Q_s + f_b Q_b$$

Q_s = Suspended load transport
Q_b = Bed load transport
Q_t = Total load transport
f_s = Suspended scaling factor
f_b = Bed scaling factor

- Larger scaling factors produce larger sediment loads and therefore larger morphology changes
- It is one of the most important parameters (driving force)
  - Controls largely the magnitude and distribution of the sediment concentration field

Concentration Capacity - Continued

- Options for sediment transport capacity:
  - Lund-CIRP (ERDC/CHL CR-07-01)
    - Separate equations for suspended and bed loads
  - Van Rijn (J. Hydraulic Eng. 2007)
    - Separate equations for suspended and bed loads
  - Watanabe (Proc. Coastal Sediments 1987)
    - One equation for total load
- These equations represent the sediment transport under equilibrium conditions

Adaptation Coefficient

- Related to how much time/distance it takes to reach equilibrium
- The larger the coefficient the more rapid the system goes into equilibrium and the larger the erosion and deposition

$$E - D = \alpha\ w_f (C_{t*} - C_t)$$

- Formulas
  - Suspended Load
    - Lin (1984)

$$\alpha = 3.25 + 0.55 \ln\left(\frac{\omega_f}{\kappa U_{*c}}\right) \qquad L_s = \frac{U_c\ d}{\alpha\ w_f}$$

    - Amanini and Silvio (1986)

$$\frac{1}{\alpha} = \frac{b}{d} + \left(1 - \frac{b}{d}\right)\exp\left[-1.5\left(\frac{b}{d}\right)^{-1/6}\frac{w_f}{U_*}\right] \qquad b = 33 z_o$$

  - Bed Load

$$L_b = \begin{cases} A_{given} d \\ L_{b,given} \end{cases}$$

  - Total Load

$$\alpha = \frac{U_c\ d}{L_t\ \omega_f} \qquad L_t = \begin{cases} (1 - r_s)L_b + r_s L_s \\ \max(L_b, L_s) \longrightarrow \text{Most Stable} \\ L_{t,given} \end{cases}$$

Bed-slope Coefficient D_s

- Bed change equation

$$(1-p_m^{'})\frac{\partial h}{\partial t}=\alpha\omega_f(C_{t*}-C_t)+\frac{\partial}{\partial x}\left[D_s(1-r_s)d\left|U\right|C_t\frac{\partial h}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_s(1-r_s)d\left|U\right|C_t\frac{\partial h}{\partial y}\right]$$

- Method first adapted by Watanabe (1985)
- Smoothes bathymetry
- Improves stability
- Related to sediment properties and flow characteristics
- Reported values
  - Watanabe (1985) used D_s = 10
  - Larson et al. (2003) and Karambas (2003) used D_s = 2
- Recommended values: 1-10

Sediment Diffusivity Coefficient

- Coefficient is related to the strength of horizontal mixing in a depth-averaged sense
- Directly related to eddy viscosity

$$K = \frac{\nu}{\sigma_s}$$

K = Diffusion coefficient
ν = Eddy viscosity
σ_s = Smidth number

- Eddy viscosity $\nu = (1-\theta_m)\nu_t + \theta_m \nu_w$
  - Falconer $\nu_t = \frac{1}{2}\left[1.15gd\frac{|U_c|}{C^2}\right]$
  - Subgrid model $\nu_t = \nu_0 + \sqrt{(\alpha_0 u_{*c} d)^2 + (l_h^2 |\bar{S}|)^2}$
  - Waves $\nu_w = \Lambda u_m H \qquad \theta_m = \left(\frac{H}{d}\right)^3$

Λ = empirical coeff.
u_m = Wave orbital velocity
H = Wave height

Total-load Correction Factor

- Accounts for the lag between the depth-averaged sediment and flow velocities
- For total load transport

$$\beta_t = \frac{1/U_c}{r_s / U_{sed} + (1 - r_s)/u_b} \qquad U_{sed} = \frac{\int_{z_b+a}^{z_s} u_s c dz}{\int_{z_b+a}^{z_s} c dz}$$

Velocity profile

$$u_s(z) = \frac{u_{*c}}{\kappa} \ln\left(\frac{z}{z_0}\right)$$

Concentration profile

$$c(z) = c_a \exp\left\{-\frac{w_f}{\varepsilon}(z-a)\right\}$$

$$\boxed{\beta_t = f\left(\frac{z_o}{d}, \frac{w_f d}{\varepsilon}\right)}$$

For sands $\beta_t \approx 0.7$

Hard-Bottom Problem

- Concentration capacity at hard-bottom cells

$$C_{t*,hb} = \min(C_t, C_{t*}) \quad \leftarrow$$ **Only allows deposition**

- Bed change equation

$$(1 - p'_m)\frac{\partial h}{\partial t} = \alpha_t \omega_f \left[\min(C_t, C_{t*}) - C_t\right] + S_{hb}$$

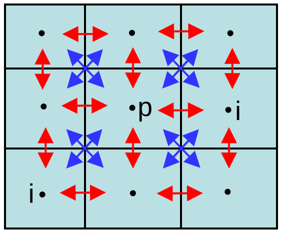
- Advection-diffusion equation

$$\frac{\partial}{\partial t}\left(\frac{dC_t}{\beta_t}\right) + \frac{\partial(dU_x C_t)}{\partial x} + \frac{\partial(dU_y C_t)}{\partial y} = \frac{\partial}{\partial x}\left[K_x d \frac{\partial(r_s C_t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[K_y d \frac{\partial(r_s C_t)}{\partial y}\right] + \alpha_t \omega_f \left[\min(C_t, C_{t*}) - C_t\right]$$

Avalanching

- Avalanching process is simulated as

$$\frac{(h_i + \Delta h_i) - (h_p + \Delta h_p)}{\Delta l} = \gamma \tan \phi_r$$



h = depth

Δl = distance between cell nodes

ϕ_r = Repose angle

$$\gamma = \begin{cases} 1 & \text{for } h_p < h_i \text{ (upslope)} \\ -1 & \text{for } h_p > h_i \text{ (downslope)} \end{cases}$$

- Two approaches available

1. 9-point mass balance approach (Wu 2007)
    - May be used at large time intervals
    - Iterates until convergence
2. Relaxation method (5- and 9-point)
    - Requires smaller time steps (morphologic time step)
    - No iterations
    - Very simple and stable

Avalanching: Relaxation Method

- Mass balance between neighboring cells

$$A_i \Delta h_i + A_p \Delta h_p = 0$$

- Repose slope condition

$$\frac{(h_i + \Delta h_i) - (h_p + \Delta h_p)}{\Delta l} = \gamma \tan \phi_r \qquad \tan \phi = \frac{h_i - h_p}{\Delta l}$$

- Depth change due to avalanching

$$\Delta h_p = R \frac{A_i \Delta l \left(\tan \phi - \gamma \tan \phi_r \right)}{A_p + A_i} \qquad \Delta h_i = -\frac{\Delta h_p A_p}{A_i}$$

$$R = \text{Relaxation factor} \approx 0.1\text{-}0.2$$

Numerical Methods

- Finite volume method
- Time integration
  - Explicit Euler scheme
- Advection
  - Upwind
  - Hybrid Linear/Parabolic Approximation (HLPA)
- Diffusion
  - Central difference
- Bed-slope term (conc.)
  - Central difference

- Boundary conditions
  - Ocean boundaries
    - Inflow: “Zero-gradient” BC
    - Outflow: Open BC
  - Land boundary
    - Zero flux BC
  - Future versions will have
    - User specified concentration (river)
    - Equilibrium BC

Questions?