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Random Wave Runup Height on Gentle Slope

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Abstract: An extensive series of labratory tests were conducted and these tests led to the development of a formula to predict runup elevation of random waves on gentle, smooth and impermeable slopes, as a function of surf similarity parameter. On gentle slopes, a bore advancing into the shoreline cannot run up when the back-rush of a preceding bore is large or when it is overtaken and captured by a subsequent large bore. No correspondence between individual running-up bores and runup crests can be seen; the number of runup crests is reduces compared to the number of running-up bores. This paper also proposed a formula for the ratio of the number of runup crests to that of incident waves; the formula can be used to estimate the mean repetition period of runup crests. The formulas proposed here are applicable to slopes, tan θ, ranging from 1/30 to 1/5 and to deep water significant wave steepness ranging from 0.007 to 0.07.

Introduction

Wave runup on coastal structures, such as seawalls and dykes, is an important factor in determining the heights of the structures; therefore, many experimental studies on wave runup for steep slopes have been carried out. Theoretical and numerical studies on wave runup have been done initially by Shen and Meyer (1963) and Freeman and Le Mehaute (1964), recently by Kobayashi et al. (1987). Wave runup and rundown on a natural beach are responsible for sand movement in the swash zone, and the maximum runup level is the limit of onshore side for on-offshore and littoral sand transport.

There are mainly two methods to analyze the characteristics of measured runup oscillations (shoreline oscillations) of random waves: one is the individual runup wave analysis; the other is the spectral analysis. From engineering viewpoints, such as determining the heights of coastal structures and artificially nourished beaches, the individual runup wave analysis is preferable, because frequency distributions or extreme value statistics of runup heights are required. The spectral analysis is employed to study the dynamic response between incident waves and runup oscillations and the spectral characteristics of runup oscillations themselves (Mase 1988). In this paper the individual runup wave analysis was employed.

Recently, in Japan, seawalls and dykes with gentle slope or artificial reefs which consist of offshore submerged breakwaters and artificially nourished beaches are recommended and constructed for the following reasons: gentle slope structures have less toe scour problems than steep slope structures; people prefer a good view and utilization around coastal zone (water front zone) as rest places. Though Huntly et al. (1977), Guza and Thornton (1982), and Holman (1986) have examined spectral characteristics or extreme value statistics of wave runup on natural beaches, studies on random wave runup

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for gentle slopes are relatively few as compared to those for steep slopes.

The purpose of this paper is to establish an experimental formula for representative runup heights of random waves on gentle, smooth and impermeable uniform slopes ranging from 1/30 to 1/5 under various wave conditions. This paper also proposed an empirical formula for the ratio of the number of runup crests to the number of incident waves. This formula can be used to estimate the mean repetition period of runup crests.

Experiments

Experiments on runup oscillation were conducted in a wave flume of 50 cm wide, 27 m long, and 75 cm deep. The test slopes of model beach, tan θ, were 1/5, 1/10, 1/20, and 1/30. The water depth in a uniform section of the flume was 45 cm for 1/5, 1/10 and 1/20 slopes, and 43 cm for 1/30 slope. Experimental setup has already been shown in Fig. 1 of Mase and Iwagaki (1984). A wave gauge of capacitance type was used as the runup meter, which was installed in a 3 cm wide and 1 cm deep groove along the center of the slope to keep a 2.2 mm diameter and 2 m long capacitance wire of the runup meter at the height of the slope surface. The runup meter was calibrated by moving every 10 cm along the slope surface (static calibration); the calibration curve was approximated by a straight line. Additional calibration of the runup meter was done by comparing runup heights of monochromatic waves measured by the runup meter with those measured by a scale (dynamic calibration). The result has been shown in Fig. 2 of Mase and Iwagaki (1984).

Random waves of the Pierson-Moskowitz type spectrum were generated in the flume. Wave groupiness was changed by two so as to have groupiness factors of 0.74 (this case is referred to Case 1) and 0.53 (Case 2). Groupiness factor is one of parameters representing a magnitude of wave grouping defined as a variation coefficient of smoothed instantaneous wave energy history (Funke and Mansard 1979). Peak frequency was changed to 0.4 Hz, 0.5 Hz, 0.6 Hz, 0.8 Hz, 1.0 Hz, and 1.2 Hz. Significant wave height was changed by three for the random waves with peak frequency from 0.4 Hz to 0.8 Hz and by two for the random waves with 1.0 Hz peak frequency and 1.2 Hz peak frequency (only for Case 1). A series of 30 runs were carried out for each slope. Water surface variations and runup oscillations were recorded simultaneously by an analog data recorder, and the records were digitized by an A-D converted at a sampling interval of 0.04 sec.

Experimental Results

Individual runup heights, R_i, were defined as the heights of crests, measured vertically from the still water level, of runup oscillations, as shown in Fig. 1. The following representative runup heights were obtained from the individual runup heights: R_{max} = the highest runup height during each run; R_2 = the 2% excess runup height; $R_{1/10}$ = the one-tenth highest runup height (the average of highest one-tenth of the total runup heights); $R_{1/3}$ = the one-third highest runup height; $\bar{R}$ = the mean runup height (the average of the total runup heights).

According to the dimensional analysis, a runup height, R, on a slope is expressed as follows (Tsuchiya et al. 1978):

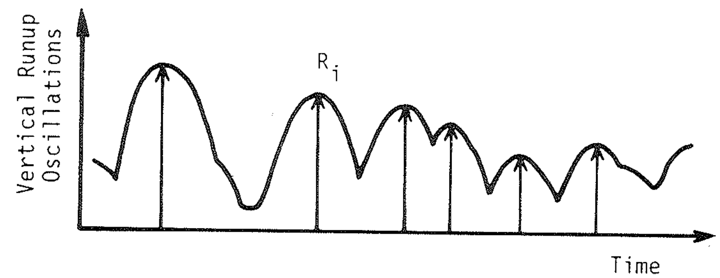


FIG. 1. Diagram of Individual Runup Height

$$\frac{R}{H} = f_1\left(\frac{H}{L}, \tan\theta, \frac{h}{H}, \frac{d}{H}, \frac{\sqrt{K}}{H}\right) \quad (1)$$

where H = a wave height; L = a wave length; tan θ = the beach slope; h = the water depth at the toe of the slope; d = the roughness height; and K = the permeability with unit of square meters, which is related to the intrinsic permeability, k, with unit of meter per second as

$$K = \frac{k\nu}{g} \quad (2)$$

where ν = the kinematic viscosity, and g = the acceleration of gravity. When h/H is larger than 3.0, the effect of h/H is negligible in monochromatic waves (Saville 1956), which is expected to the case of random waves; thus, for a smooth and impermeable slope, Eq. 1 becomes

$$\frac{R}{H} = f_2\left(\frac{H}{L}, \tan\theta\right) \quad (3)$$

Hunt (1959) proposed the following equation, based on experimental data, for runup heights of monochromatic waves breaking on the slope:

$$\frac{R}{H} = \frac{\tan\theta}{\sqrt{\frac{H}{L}}} \quad (4)$$

In this study, representative runup heights of random waves were examined based on Eq. 3, in which the deep water significant wave height, H_0, and the deep water significant wave length, L_0, calculated from the significant wave height and period at the deepest measuring point, are taken as H and L. Table 1 shows laboratory experimental data of H_0, H_0/L_0, surf similarity parameter ξ (= $\tan\theta/\sqrt{H_0/L_0}$) (Battjes 1974), representative runup heights normalized by H_0, and the ratio α of the number of runup crests to that of incident waves.

Mase and Iwagaki (1984) plotted the R_{max}/H_0, $R_{1/3}/H_0$, and $\bar{R}/H_0$ against the deep water significant wave steepness, H_0/L_0, on a log–log scale. The gradient of straight lines representing the tendency of the experimental data of $H_0/L_0 \geqq 0.007$ was −0.37, which is different from −0.5 given by Eq.

TABLE 1. Laboratory Experimental Data

H_0 (1)	H_0/L_0 (2)	ξ (3)	R_{max}/H_0 (4)	R_2/H_0 (5)	$R_{1/10}H_0$ (6)	$R_{1/3}H_0$ (7)	$\bar{R}/H_0$ (8)	α (9)
(*a*) 1/5 Beach Slope (Case 1)								
5.96	0.007	2.44	4.04	3.31	2.97	2.36	1.46	0.95
4.84	0.006	2.73	4.29	3.34	2.97	2.42	1.49	0.99
3.92	0.005	3.02	4.24	3.29	3.02	2.44	1.46	0.98
7.03	0.014	1.72	3.57	3.05	2.76	2.23	1.46	0.81
5.94	0.011	1.91	4.10	3.02	2.77	2.33	1.51	0.82
4.53	0.009	2.20	4.13	3.28	3.13	2.63	1.66	0.87
8.89	0.023	1.34	2.77	2.32	2.10	1.69	1.09	0.79
6.88	0.019	1.49	3.47	2.55	2.28	1.85	1.18	0.83
5.17	0.014	1.74	3.79	2.80	2.54	2.06	1.31	0.83
10.85	0.048	0.92	2.25	1.82	1.66	1.33	0.87	0.67
8.61	0.040	1.02	2.56	1.93	1.77	1.44	0.94	0.69
6.36	0.031	1.16	2.77	2.23	2.05	1.64	1.06	0.71
6.86	0.049	0.92	2.23	1.78	1.63	1.29	0.85	0.67
5.66	0.041	1.00	3.11	1.92	1.75	1.36	0.87	0.66
6.93	0.060	0.83	1.75	1.46	1.29	1.00	0.62	0.67
6.12	0.056	0.86	2.55	1.48	1.31	1.01	0.62	0.68
(*b*) 1/5 Beach Slope (Case 2)								
6.16	0.007	2.41	3.95	3.12	2.86	2.32	1.49	0.93
4.86	0.005	2.76	4.70	3.24	2.92	2.41	1.53	0.97
4.21	0.005	2.93	4.67	3.20	2.96	2.44	1.58	0.98
7.48	0.015	1.68	3.35	2.90	2.64	2.16	1.43	0.81
5.90	0.011	1.91	3.76	3.06	2.82	2.39	1.54	0.83
5.07	0.010	2.06	3.81	3.14	2.99	2.54	1.61	0.84
9.29	0.024	1.30	2.68	2.29	2.10	1.71	1.13	0.78
6.96	0.019	1.47	2.96	2.56	2.37	1.99	1.31	0.78
5.60	0.015	1.65	3.36	2.82	2.61	2.20	1.44	0.80
11.08	0.049	0.91	2.20	1.82	1.65	1.40	0.94	0.67
8.61	0.041	1.01	2.72	1.93	1.78	1.49	0.98	0.71
7.29	0.035	1.09	2.50	2.05	1.91	1.59	1.08	0.71
7.41	0.052	0.89	2.08	1.76	1.67	1.39	0.89	0.68
5.73	0.043	0.98	2.39	2.06	1.90	1.51	0.98	0.66
(*c*) 1/10 Beach Slope (Case 1)								
5.16	0.007	1.23	2.76	2.26	2.05	1.61	0.95	0.72
3.96	0.005	1.40	3.16	2.43	2.25	1.75	1.02	0.77
2.94	0.004	1.65	3.42	2.74	2.47	1.88	1.09	0.78
6.79	0.015	0.83	2.10	1.67	1.46	1.15	0.72	0.62
5.37	0.012	0.93	2.25	1.71	1.59	1.25	0.79	0.62
3.96	0.009	1.08	2.39	1.93	1.72	1.35	0.83	0.67
8.72	0.024	0.64	1.72	1.24	1.16	0.95	0.62	0.58
6.81	0.020	0.71	1.83	1.40	1.29	1.02	0.65	0.61
5.03	0.014	0.84	1.88	1.52	1.39	1.06	0.67	0.67
11.02	0.049	0.45	1.24	1.12	0.99	0.79	0.51	0.48
8.93	0.041	0.49	1.49	1.13	1.03	0.82	0.53	0.50
6.72	0.032	0.56	1.71	1.13	1.05	0.81	0.53	0.54
6.79	0.049	0.46	1.35	1.11	0.97	0.76	0.50	0.48
5.65	0.041	0.49	1.32	1.10	1.02	0.81	0.54	0.46

TABLE 1. (*Continued*)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
6.94	0.060	0.41	1.29	0.97	0.86	0.68	0.44	0.44
5.94	0.054	0.43	1.26	0.96	0.86	0.68	0.44	0.47
(*d*) 1/10 Beach Slope (Case 2)								
5.31	0.007	1.24	2.52	2.08	1.93	1.60	1.01	0.75
3.90	0.005	1.42	2.85	2.33	2.18	1.77	1.10	0.77
3.16	0.004	1.57	3.06	2.58	2.38	1.91	1.19	0.73
7.21	0.015	0.83	1.85	1.46	1.36	1.12	0.74	0.65
5.39	0.011	0.95	2.01	1.66	1.53	1.24	0.80	0.68
4.54	0.009	1.04	2.11	1.81	1.63	1.31	0.84	0.71
9.16	0.025	0.63	1.52	1.22	1.14	0.96	0.64	0.59
6.97	0.020	0.71	1.57	1.33	1.24	1.04	0.69	0.62
5.75	0.017	0.78	1.74	1.47	1.36	1.10	0.72	0.64
11.08	0.050	0.45	1.28	1.04	0.91	0.74	0.50	0.49
8.80	0.042	0.49	1.17	1.03	0.94	0.76	0.50	0.53
7.21	0.035	0.54	1.24	1.10	0.99	0.80	0.52	0.52
7.40	0.053	0.43	1.14	0.91	0.85	0.66	0.43	0.50
5.68	0.043	0.48	1.16	0.99	0.88	0.71	0.47	0.49
(*e*) 1/20 Beach Slope (Case 1)								
4.77	0.006	0.65	1.68	1.38	1.24	1.01	0.61	0.55
3.68	0.005	0.73	1.84	1.41	1.34	1.07	0.63	0.58
2.69	0.004	0.85	2.01	1.63	1.51	1.14	0.69	0.57
6.39	0.013	0.44	1.28	1.10	1.00	0.80	0.49	0.42
4.92	0.010	0.49	1.43	1.16	1.06	0.83	0.52	0.44
3.62	0.008	0.58	1.49	1.24	1.09	0.82	0.50	0.50
7.93	0.021	0.35	1.05	0.87	0.81	0.67	0.45	0.37
6.15	0.017	0.39	1.20	0.90	0.85	0.71	0.43	0.41
4.50	0.012	0.45	1.30	0.95	0.88	0.69	0.42	0.48
9.99	0.044	0.24	0.82	0.59	0.56	0.48	0.31	0.33
7.87	0.036	0.26	0.87	0.64	0.62	0.50	0.32	0.37
5.82	0.027	0.31	0.94	0.78	0.68	0.53	0.35	0.39
6.93	0.049	0.23	0.76	0.63	0.61	0.48	0.31	0.30
5.73	0.043	0.24	0.73	0.66	0.60	0.48	0.31	0.31
7.00	0.062	0.20	0.68	0.63	0.56	0.41	0.27	0.31
6.00	0.056	0.21	0.67	0.62	0.54	0.42	0.27	0.30
(*f*) 1/20 Beach Slope (Case 2)								
4.95	0.006	0.64	1.56	1.21	1.12	0.92	0.57	0.58
3.62	0.005	0.74	1.79	1.28	1.21	0.97	0.60	0.61
2.93	0.004	0.82	1.85	1.37	1.28	1.01	0.63	0.64
6.68	0.013	0.44	1.20	1.10	0.97	0.78	0.50	0.45
5.04	0.010	0.50	1.27	1.15	1.02	0.82	0.52	0.47
4.04	0.008	0.56	1.29	1.18	1.05	0.82	0.52	0.51
8.37	0.023	0.33	0.86	0.81	0.72	0.61	0.40	0.40
6.19	0.017	0.38	0.85	0.81	0.76	0.63	0.42	0.45
5.14	0.014	0.42	0.92	0.86	0.79	0.64	0.41	0.47
10.16	0.046	0.23	0.74	0.64	0.60	0.49	0.33	0.32
7.72	0.036	0.26	0.80	0.69	0.66	0.56	0.38	0.33
6.35	0.030	0.29	0.86	0.78	0.71	0.58	0.38	0.36
7.35	0.053	0.22	0.73	0.64	0.59	0.45	0.30	0.32
5.78	0.045	0.24	0.71	0.65	0.60	0.48	0.31	0.34

TABLE 1. (Continued)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(g) 1/30 Beach Slope (Case 1)								
4.69	0.006	0.43	1.15	1.15	1.04	0.85	0.53	0.38
3.56	0.005	0.49	1.47	1.18	1.09	0.89	0.56	0.42
2.64	0.004	0.56	1.47	1.27	1.17	0.91	0.55	0.45
6.09	0.013	0.29	0.92	0.89	0.81	0.68	0.41	0.32
4.76	0.010	0.33	1.10	0.89	0.83	0.71	0.43	0.34
3.45	0.007	0.40	1.11	0.95	0.88	0.71	0.40	0.42
7.66	0.021	0.23	0.74	0.74	0.73	0.60	0.40	0.28
5.91	0.016	0.26	0.96	0.91	0.79	0.63	0.41	0.31
4.75	0.014	0.28	1.12	0.80	0.76	0.60	0.38	0.34
9.99	0.047	0.15	0.57	0.56	0.51	0.42	0.28	0.24
7.83	0.036	0.17	0.65	0.54	0.52	0.44	0.29	0.26
5.72	0.027	0.20	0.63	0.58	0.55	0.45	0.30	0.31
6.58	0.050	0.15	0.51	0.48	0.45	0.38	0.26	0.24
5.36	0.041	0.16	0.55	0.51	0.47	0.40	0.27	0.24
6.84	0.065	0.13	0.48	0.46	0.42	0.36	0.22	0.22
5.88	0.058	0.14	0.50	0.49	0.45	0.36	0.24	0.22
(h) 1/30 Beach Slope (Case 2)								
4.75	0.006	0.43	1.19	1.00	0.92	0.78	0.51	0.42
3.47	0.004	0.51	1.34	1.03	0.97	0.82	0.52	0.48
2.87	0.004	0.56	1.32	1.10	1.00	0.81	0.50	0.53
6.19	0.012	0.31	0.92	0.89	0.78	0.65	0.42	0.35
4.70	0.009	0.35	1.09	0.89	0.83	0.69	0.42	0.38
3.84	0.007	0.39	1.10	0.91	0.87	0.68	0.40	0.42
8.11	0.022	0.22	0.69	0.69	0.64	0.55	0.37	0.31
6.09	0.017	0.26	0.85	0.71	0.68	0.58	0.38	0.32
5.00	0.013	0.29	0.87	0.76	0.71	0.60	0.38	0.36
9.99	0.046	0.15	0.55	0.54	0.50	0.40	0.28	0.24
7.49	0.036	0.18	0.73	0.57	0.54	0.46	0.32	0.22
6.34	0.031	0.19	0.68	0.61	0.55	0.48	0.32	0.26
7.00	0.053	0.15	0.49	0.46	0.44	0.36	0.23	0.25
5.51	0.043	0.16	0.59	0.55	0.47	0.39	0.25	0.26

4. In the cases of $H_0/L_0 \leqq 0.005$ for 1/20 and 1/30 slopes, the observations of $R_{1/3}/H_0$ and $\bar{R}/H_0$ were a little smaller than the predictions by the straight lines. In particular, for 1/5 slope, nondimensional representative runup heights were considerably smaller than the predictions by the straight lines in the region of $H_0/L_0 < 0.007$. Although the same electric signal was used to generate random waves, the incident significant wave height in the case of 1/5 slope was larger than that measured in other gentler slopes, due to wave reflection at the slope, when the wave steepness is very small (see Table 1). Larger values of H_0 causes smaller values of R_{max}/H_0, $R_{1/3}/H_0$, and $\bar{R}/H_0$. Part of the reason that some of the data with very small wave steepness do not follow the tendency of the straight line may be attributed to the surging or collapsing breaker conditions.

Ahrens (1979) investigated random wave runup on 1/1.5 slope, and showed that the larger the wave steepness becomes the larger the nondimensional runup height becomes. This tendency is quite opposed to the present result for gentle slopes; which depends on the conditions of breaking or non-breaking. For breaking random waves, the nondimensional runup heights become small as the wave steepness becomes large.

Concerning the relationship between the nondimensional runup heights and the beach slope, Mase and Iwagaki (1984) reported that the nondimensional runup heights become large as the beach slope becomes steep. This tendency is opposed to those measured by Ahrens (1983) and Kamphuis and Mohamed (1978) for steep slopes ranging from 1/4 to 1/1. For breaking random waves, the nondimensional runup height is proportional to the beach slope.

When the slope is gentle, a bore advancing into the shoreline cannot run up when the back-rush of a preceding bore is large, or when it is overtaken and captured by a subsequent large bore before the running-up bore reaches a maximum runup level, because periods of one cycle of run-up and run-down become large compared to those of incident waves. Therefore, the number of runup crests is reduced compared to that of incident waves, and no correspondence between individual running-up bores and runup crests can be seen (Mase 1988a). In addition, low frequency components become predominant in runup spectra (Mase 1988a).

The ratio of the number of runup crests to that of incident waves, α, can be well arranged by the surf similarity parameter, ξ, shown in Fig. 8 of Mase and Iwagaki (1984). As the ξ becomes large, the α approaches 1.0; this tendency is similar to that of Ahrens (1983) for steep slopes. There is a method to estimate the frequency distribution of runup heights of random waves which assumes that the runup height due to each incident individual wave is equal to the runup height of a corresponding monochromatic wave with same wave height and period as those of the individual wave; the method cannot be applicable to the case of small ξ.

It was demonstrated by Carstens et al. (1966) and Johnson et al. (1978) that random waves with remarkable wave grouping cause larger runup. However, for gentle slopes, there is little effect of wave grouping of incident waves on representative runnup heights and on the ratio α, because the difference of wave grouping of incident waves disappears under the shoreline through wave breaking (Mase and Iwagaki 1984; Mase 1988b).

Experimental Formulas for Random Wave Runup on Gentle Slope

Representative Runup Heights

The following formula is proposed for representative runup heights of random waves on gentle, smooth and impermeable slopes:

$$\frac{R}{H_0} = a\xi^b, \quad \text{for } \frac{1}{30} \leqq \tan\theta \leqq \frac{1}{5} \quad \text{and } 0.007 \leqq \frac{H_0}{L_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

The coefficients in Eq. 5 were determined by the least squares method, except for the experimental data of which deep water significant wave steepnesses are smaller than 0.007 for 1/5 slope and smaller than 0.005 for 1/20 and 1/30 slopes. The coefficients are

$a = 2.32, \quad b = 0.77, \quad$ for R_{max};

$a = 1.86, \quad b = 0.71, \quad$ for R_2;

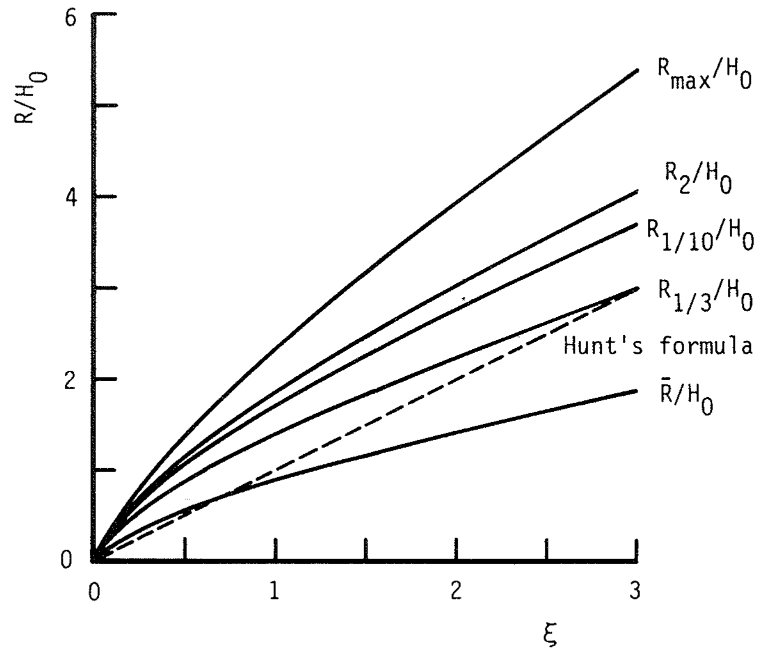


FIG. 2. Proposed Runup Height Formula for Breaking Random Waves

$a = 1.70,\quad b = 0.71,\quad$ for $R_{1/10}$;

$a = 1.38,\quad b = 0.70,\quad$ for $R_{1/3}$;

$a = 0.88,\quad b = 0.69,\quad$ for $\bar{R}$. . . . . (6)

Fig. 2 shows the result of Eq. 5 with the coefficients denoted by Eq. 6 as well as Eq. 4 by Hunt (1959). Hunt's result lies between curves of $R_{1/3}H_0$ and $\bar{R}/H_0$.

Fig. 3 shows the comparisons of observations and predictions of representative runup heights. Both measured and predicted values agree well in all five figures. The correlation coefficient (C.C.) between both values is 0.98 for Fig. 3(*e*), and 0.99 for the other cases, and the standard deviations (S.D.) between both values are 0.16, 0.09, 0.09, 0.09, and 0.07 for Fig. 3(*a*), (*b*), (*c*), (*d*), and (*e*), respectively.

The predictions given by Eq. 5 are substantially larger than the observations on a natural beach of which slope is about 1/10 by Holman (1986). Eq. 5 gives an envelope for the maximum of the scattered observations. To describe an average trend of the observations, the right hand side of Eq. 5 must be multiplied by 0.5. The difference between the observations measured on a natural beach and the predictions by Eq. 5 seems to depend on the differences in permeability and roughness.

Ratio of Number of Runup Crests to Incident Waves

When the ratio α is plotted against the surf similarity parameter ξ on a

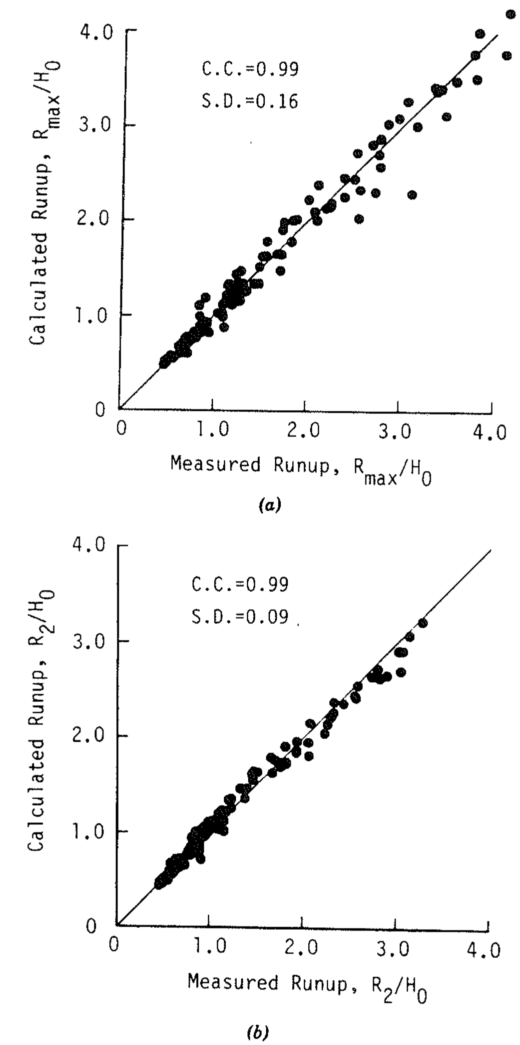
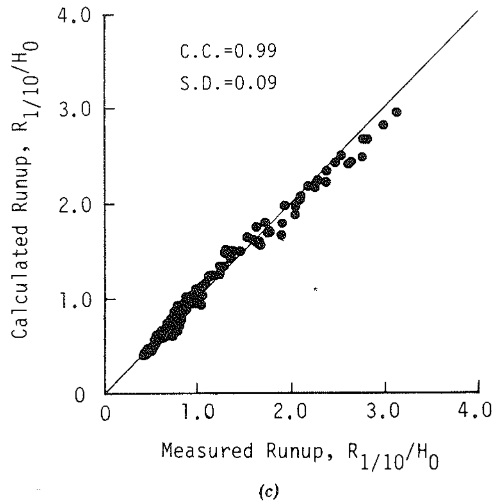


FIG. 3. Comparison of Observations and Predictions of Runup Heights: (*a*) R_{max}/H_0; (*b*) R_2/H_0

log–log scale, the characteristic of the change is represented by a convex curve (see Fig. 8 of Mase and Iwagaki 1984). However, in this paper, the straight line which changes its slope at some point was adopted for simplicity. Firstly, two straight lines were obtained by the least squares method using the experimental data of $\xi \geqq 1$ and of $\xi < 1$, respectively. Subsequently, the intersection of the two regression lines and that of the regression line for $\xi \geqq 1$ and the line of $\alpha = 1$ were calculated; the values of ξ at the intersections were 0.91 and 3.57, respectively. The final result is described by



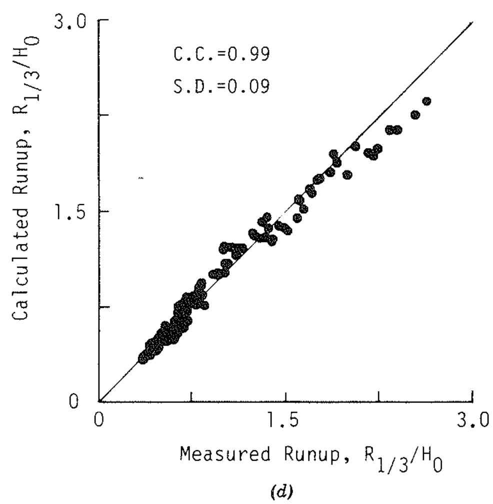


FIG. 3. Comparison of Observations and Predictions of Runup Heights: (*c*) $R_{1/10}H_0$; (*d*) $R_{1/3}H_0$

$$\alpha = 0.72\xi^{0.58}, \qquad \text{for } \xi \leqq 0.91$$

$$\alpha = 0.70\xi^{0.28}, \qquad \text{for } 0.91 < \xi \leqq 3.57$$

$$\alpha = 1.0, \qquad \text{for } 3.57 < \xi \quad (7)$$

Fig. 4 shows the comparison of observations and predictions of α. Both values agree well; the correlation coefficient (C.C.) and the standard deviation (S.D.) between both values are 0.99 and 0.02, respectively.

The mean repetition period of runup crests, T_R, is estimated as follows using Eq. 7:

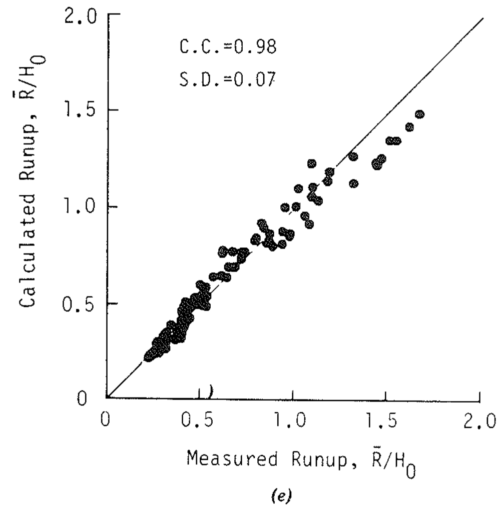


FIG. 3. Comparison of Observations and Predictions of Runup Heights: (*e*) $\bar{R}/H_0$

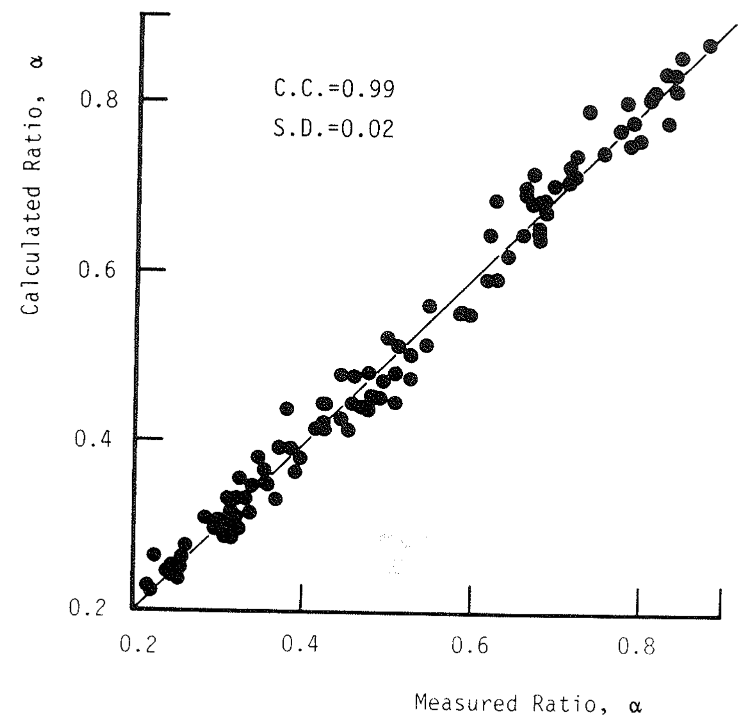


FIG. 4. Comparison of Observations and Predictions of α

$$T_R = \frac{\bar{T}}{\alpha} \qquad (8)$$

where $\bar{T}$ = the incident mean wave period.

The value of $\xi = 3.57$ in Eq. 7 corresponds well to the value in Fig. 4 of Ahrens (1983) and in Fig. 9 of Holman (1986) at which the mean zero-upcrossing runup period and the peak period of incident wave are nearly equal.

Conclusions

A total of 120 test runs of runup on gentle, smooth and impermeable slopes were conducted by using random waves. The slope ranged from 1/30 to 1/5, and deep water significant wave steepness from 0.004 to 0.07. This study proposed an experimental formula for representative runup heights of random waves normalized by the deep water significant wave height, H_0, such as R_{max}/H_0, R_2/H_0, $R_{1/10}/H_0$, $R_{1/3}/H_0$, and $\bar{R}/H_0$, as a function of surf similarity parameter. The formula is expressed by Eq. 5 with the coefficients described by Eq. 6. The tendency that the nondimensional runup heights become large as the deep water wave steepness becomes small and as the slope becomes large is opposite to that measured for steep slopes by Ahrens (1979, 1983) and Kamphuis and Mohamed (1978), which depends on the condition of breaking or non-breaking. To estimate the mean repetition period of runup crests, a formula for the ratio of the number of runup crests to that of incident waves was proposed, which is expressed by Eq. 7.

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Appendix II. Notation

The following symbols are used in this paper:

- d = roughness height;
- g = acceleration of gravity;
- H = wave height;
- H_0 = deep water significant wave height;
- H_0/L_0 = deep water significant wave steepness;
- h = water depth at toe of slope;
- K = permeability with unit of square meters;
- k = permeability with unit of meter per second;
- L = wave length;
- L_0 = deep water significant wave length;
- R = runup height;
- $\bar{R}$ = mean runup height;
- $R_{1/3}$ = one-third highest runup height;
- $R_{1/10}$ = one-tenth highest runup height;
- R_2 = 2% excess runup height;
- R_i = individual runup height;
- R_{max} = highest runup height;
- $\bar{T}$ = incident mean wave period;
- T_R = mean repetition period of runup crests;
- $\tan\theta$ = beach slope;
- α = ratio of the number of runup waves to that of offshore incident waves;
- ξ = surf similarity parameter; and
- ν = kinematic viscosity.